Capital budgeting in a situation with variable utilisation of capacity – an example from the pulp industry

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Abstract

It is well known that the profitability within the process industry is heavily dependent upon the degree of utilisation of the plants. Utilisation, in turn, is dependent upon the often very volatile market conditions for the commodity produced.

This paper examines the implications for capital budgeting, dealing with a situation of changing levels of utilisation. A paper-pulp mill is chosen for the purpose of investigating whether, in this specific case, the variation of utilisation in response to changing market conditions affects plant value in any major way.

Comparing a fixed and a variable production rate (using the net present value rule and option pricing by means of the Feynman-Kac formula), it is found that the difference in value is considerable. However, an inappropriately specified price process may explain the difference. The geometric Brownian motion assumed allows the price to decline to almost zero. In order to overcome this problem, an alternative price process allowing for mean reversion in the nominal price of pulp is developed and tested. The value of the ability to cut production is then found to be insignificant.

Based on the findings of this study, it is not worthwhile to model a variable utilisation of capacity. It is, however, of utmost importance to evaluate different assumptions about pulp price behaviour, as this will affect results substantially.

Keywords: Capital Budgeting, Real Options, Mean Reversion, Feynman-Kac

JEL classification code: C60, G31, M21

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1. Introduction

Background

As long as variable costs are non-zero, a company's profit will vary more when price varies and less when volume changes. A price increase does not invoke any extra costs, whereas an extra unit sold does. If the market is not perfect, in that a single actor can affect the market price acting alone, it makes sense for a producer to reduce production in order to reverse a price decline. This is especially so in the process industry where capacity expansion is slow. It takes time for the competitors to gain market shares. Besides, existing competitors are presumably equally interested in keeping the price up. Even without producers forming a cartel, which would violate antitrust laws, we may well observe behaviour where producers reduce production in times of heavy downward pressure on the price.

The latter may well be described as company policy for the major pulp and paper producer STORA, nowadays Stora Enso, who in their annual reports both -95 and -96 states that: *"Price changes have more than double the effect on income compared with volume changes, as a result of which STORA gives priority to maintaining prices in a weakening market compared with unchanged production volume."* [1]

This declaration finds support in the industry statistics. It is hardly surprising to find that during periods of high price, the utilisation of capacity has been high and vice versa. The change in aggregate production volume is in the order of 10-20%.[2]

There are several good reasons to reduce pulp production in response to a price decline. One is to cut down on the storage levels of pulp. A price decline is normally preceded by an excessive supply compared with demand, resulting in high storage levels and a following price pressure.

Another reason for reducing pulp production is to put pressure on the price of pulpwood. Pulpwood is, of course, the major input to pulp production. By cutting production, an excess of pulpwood is created in the market and new price negotiations will commence with the forestry owners. A fall in the price of pulpwood will most certainly be the outcome of these negotiations. A comparison of pulp- and pulpwood prices shows a correlation of 0.74 for the period 1980-1996. An additional benefit of reducing production is the possibility to cut down on the most expensive or, when it comes to quality, inferior pulpwood first. Due to the amount of pulpwood consumed in a mill there are often logistic problems, getting access to the amount needed. The plants have to be supplied from forests further and further away, resulting in increasing costs. By reducing output, the marginal cost of pulpwood is also reduced.

Research issue

[1] STORA annual report 1996, page 13.

[2] Source: Hansson & Partners database Ecowin, "Swedish production paper and paperboard volume".

The question is if the willingness to decrease production in periods of low or declining price should affect the practice of capital budgeting? Obviously, it should, if the value of operating a variable production policy deviates substantially from using a fixed production rate.

However, establishing the plant value under a variable production policy is more difficult than it may first appear. Often, in practical capital budgeting, many different sources of cash flow are treated as equally risky and a single risk adjusted discount rate is used. As is pointed to in standard corporate finance textbooks, this is a simplification. Each cash flow should be considered separately and discounted with an interest rate appropriate to its systematic risk. Given a fixed production rate, valuation of cash flow resulting from sales of pulp is straightforward: Calculate, by means of the CAPM or any other market equilibrium model, the required rate of return for holding pulp and discount the cash flow accordingly.

Under a variable production rate, however, this procedure breaks down. Cash flow stemming from pulp sales now becomes a convex function of pulp price. During a recession, not only is the price low, the volume is also below normal. As a result, pulp price variation is no longer a measure of the risk of the cash flow. Luckily, option theory has been developed to deal with this situation, valuation of an arbitrary contract dependent upon an underlying asset.

In fact, the cash flow sometimes resembles that of an ordinary call option. Production is maintained as long as market price exceeds costs and the cash flow is the difference between the two. If, on the other hand, costs exceed price, operation ceases and the resulting cash flow will be zero. The whole plant can then be valued as series of call options, expiring one at a time.[3]

Outline of the study

In this paper the value of a fictitious, but realistic, pulp mill will be calculated using two techniques. First, a net present value calculation is applied on a plant capable of producing 400 000 tons of pulp annually. Thereafter, we will arrive at the same result using a real option technique. At this stage, a situation with varying utilisation of capacity will be introduced. Through this three-step approach, it is possible to isolate the effect of changing the scale of production from other effects, parameter settings, different assumptions etc., that may affect the result. Having introduced the option framework, further comparisons are made. This time by changing the stochastic process that the pulp price is assumed to follow.

The choice of pulp production as a case study was natural. It is both convenient and important. The convenience stems from pulp being a traded commodity with an established market price, thereby readily allowing derivative pricing. As a mature business, with not many options attached to production, modelling can be simplified without deviating too much from reality. Being a large part of the forestry industry, pulp production is also important to the Swedish economy. Of the total trade balance surplus of 131 billion SEK in 1997, 76 came

[3] Several authors have been credited for being the first to recognise the similarity between a call option and the cash flow from operating a plant. McDonald and Siegel (1985) is one of the best, but probably not the first example.

from forestry industry products.[4] In Sweden, investments in the forestry sector have long-ranging economic consequences even outside the industry. Consequentially, even the procedure of capital budgeting has economic significance.

Approach

Let the stochastic way in which the pulp price evolves over time be described by an Ito process. As pulp is a traded commodity, the arbitrage free value of the plant *V(t,P)*, is a solution to the famous Black and Scholes differential equation:

$$\frac{1}{2}\sigma^2 P^2 V_{PP} + (r-\delta) P V_P + V_t - rV + \Pi(t,P) = 0\,,$$

where $\Pi(t,P)$ is the flow of payments generated by the plant.

As will be shown later, the above differential equation can be solved using a technique developed by Feynman-Kac. The solution is:

$$V(t_0,P) = e^{-r(T-t_0)} E^Q\left[V(T,P)\right] + \int_{t_0}^{T} e^{-r(t-t_0)} E^Q\left[\Pi(t,P)\right] dt\,.$$

The above formula seems rather complicated at first glance, but a more careful look should reveal the intuition. The value of the plant today, equals the present value of the salvage value plus the present value of all payments generated through operation of the plant. The derivation of the Feynman-Kac formula is provided in section 2.

Section 3 presents market and plant data as well as parameter settings. In sections 4 and 5 plant value is established for a fixed- and variable production policy, respectively.

Under the assumption of a geometric Brownian motion, a net present value calculation can be seen as a special case of the Feynman-Kac formula. However, the formula is applicable to any Ito process, not only the geometric Brownian motion. As an alternative to the random walk, we will in section 6 model the pulp price as mean reverting, by letting the logarithm of the pulp price follow an Ornstein-Uhlenbeck process. The process features an interesting property in that the reversion price is allowed to increase over time, thus enabling the nominal price to be mean reverting. This is an advantage over existing financial models, which treat the real price as mean reverting. The new model is applied to pulp data and plant value is calculated for both the fixed and variable production policies.

Section 7 contains an exposition of other real options, in addition to the variable production rate. However, these are all minor in the pulp industry. Section 8 concludes the study.

[4] Source: The Swedish Forestry Industries Association. (www.forestindustries.se)

2. Derivation of the Feynman–Kac formula

The Black and Scholes differential equation

Assume a market with no taxes or transaction costs and where no agent has private information or can exercise market power. The latter assumptions are needed to ensure that the pulp price process is exogenous and we specify it as the Ito process

$$dP = \alpha(t, P)dt + \sigma(t, P)dw. \tag{2.1}$$

The change in price dP is partly deterministic, specified by the term $\alpha(t, P)dt$, and partly stochastic. The stochastic behaviour is given by the volatility function $\sigma(t, P)$ and the Wiener-increment dw. We also assume the existence of a deterministic short rate of interest r. The short rate of interest will be held constant throughout this paper, as it will greatly simplify the notation, but all results hold as long as the short rate is a deterministic function of time.

To establish the value of the plant $V(t, P)$, consider the portfolio

$$\varnothing = V(t, P) - P \cdot V_P(t, P). \tag{2.2}$$

Where V is the value of the plant.
$-P \cdot V_P$ is the value of V_P short positions in pulp.
V_P denotes the partial derivative of $V(t, P)$ with respect to the argument P.

The portfolio's return during the small increment of time dt is,

$$d\varnothing = dV + \Pi(t,P)dt - V_P\, dP - \delta P V_P\, dt. \tag{2.3}$$

Where dV is the change in plant value during dt.
$\Pi(t,P)$ is the profit flow generated by operating the plant during dt.
$-V_P\, dP$ is the number of short positions multiplied by the change in price.
$-\delta P V_P\, dt$ is the payment that has to be made to the lender of pulp.

The term $\delta P V_P\, dt$ is worth some special attention and so is the notion of a short position. To create a short position in pulp, someone must be willing to lend the pulp, so that you can resell the borrowed pulp in the market and thereby establish the short position. Now, the question is, why should anybody, presumably a paper producer holding pulp for later manufacturing, be willing to lend you the pulp?

To answer that question, another question has to be asked. Why does the lender hold an inventory of pulp in the first place? The expected price increase is not enough to motivate the inventory, so strictly on a financial basis, it should not exist. The reason for its existence is, of course, the convenience an inventory provides. Smoothing differences in supply and demand, avoiding local shortages, enhancing scheduling flexibility etc., the end goal being to disallow interruptions in paper production.

The inventory should, obviously, be large enough to serve its purpose. But, on the other hand, not too large, as the financial- and storage costs then would be excessive. Financial economists often refer to the difference between the required rate of return (if the commodity was seen as an investment object) and the expected price increase, as the Marginal Convenience Yield net of Storage Costs, $\delta(t, P)$, or just convenience yield for short. As this is what the lender gives up, it is also what he should be compensated for. Thereby the term $-\delta P V_P\, dt$.

The required rate of return for holding pulp will be denoted by μ. Generally, μ is allowed to be any deterministic function of time, but is in this paper held constant, since this is almost exclusively, the assumption made in practice.[5] The expected price increase is given by the Ito process and can be expressed as $\frac{1}{dt}E\left[\frac{dP}{P}\right]$, i.e. the expected percentage price change over the short time interval dt.[6]

Algebraically, the relation is

$$\mu = \frac{1}{dt}E\left[\frac{dP}{P}\right] + \delta(t, P), \tag{2.4}$$

and the convenience yield $\delta(t, P)$ is often referred to as the rate of return shortfall, as it is the difference between the required return and the expected price increase, and this will be the term henceforth used in the paper. The name dividend yield is also used for financial assets. For a more thorough discussion on the topic, Mcdonald and Siegel (1984) is recommended.

Let us return to the derivation of the Black and Scholes differential equation. Applying Ito´s lemma on dV in (2.3) gives[7]

$$dV = V_t dt + V_P dP + \tfrac{1}{2} V_{PP} (dP)^2$$

where

$$(dP)^2 = (\alpha\, dt + \sigma\, dz)^2 = \sigma^2 (dz)^2 = \sigma^2 dt\,.$$

The portfolio return of (2.3) becomes $d\varnothing = (V_t - \delta\, P V_P + \tfrac{1}{2}\sigma^2 V_{PP} + \Pi)dt$. Note that this return is risk free, since the Wiener increment dw is missing. Therefore, the portfolio's return must also equal $r\,\varnothing\, dt$ in order to disallow arbitrage opportunities. We thereby get the equality

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[5] Note that μ is independent of P. The required compensation for systematic risk may well vary over time but is, naturally, independent of the price of pulp.

[6] For the geometric Brownian motion $dP = \alpha P dt + \sigma\, P dw$, the expected price increase is just the constant α. With the risk adjusted return μ held constant, the standard option textbook expression $\mu = \alpha + \delta$ is obtained, with δ being a constant.

[7] Ito´s lemma can, for all practical reasons, be seen as an ordinary Taylor series expansion dropping all higher terms and noting that $(dw)^2 = dt$.

$$(V_t - \delta\, PV_P + \tfrac{1}{2}\sigma^2 V_{PP} + \Pi)dt = r(V - PV_P)dt\,.$$

This equality must be fulfilled for all times *dt*, giving the deterministic differential equation

$$V_t + (r - \delta)PV_P + \tfrac{1}{2}\sigma^2 V_{PP} + \Pi - rV = 0\,.$$

Writing explicitly all the variables suppressed in the derivation, the Black and Scholes differential equation becomes

$$V_t(t,P) + (r - \mu + \frac{1}{dt}E\left[\frac{dP}{P}\right])PV_P(t,P) + \tfrac{1}{2}\sigma^2(t,P)V_{PP}(t,P) + \Pi(t,P) - rV(t,P) = 0\,. \qquad (2.5)$$

This is a general equation for pricing of all contracts whose value is a function of another asset, the so-called underlying asset. In this case, the value of a pulp plant is a function of the market price of pulp. In the case of a stock option, the value of the option is a function of the stock price.

Depending on the type of asset or contract that is to be valued, parameters and boundary conditions change. In the case of the famous ”Black and Scholes formula for a European call option on a non dividend paying stock”, the dividend yield $\delta(t, P)$ is equal to zero. Further, a stock option gives the owner no profit flow before maturity, so $\Pi(t,P)$ must also be zero. Finally, the payment at maturity, *max*{*stock price - exercise price* , 0}, is the boundary condition that the differential equation must satisfy.[8]

The Feynman - Kac formula

The previous differential equation can be solved through an elegant statistical technique, which we will now go through. The resulting formula is called the Feynman-Kac formula after the originators.

Start by rewriting the Black and Scholes equation (2.5) as,

$$V_t(t,P) + \kappa(t,P)PV_P(t,P) + \tfrac{1}{2}\sigma^2(t,P)V_{PP}(t,P) + \Pi(t,P) - rV(t,P) = 0\,, \qquad (2.6)$$

and assume that a new variable, also called $P(t)$, follows the diffusion process

$$dP = \kappa(t,P)Pdt + \sigma(t,P)dv\,, \qquad (2.7)$$

where dv is the increment from (another) Wiener process. Note that $\kappa(t, P)$ is defined in (2.6).

[8] Strictly speaking, in the pioneering Black and Scholes article of 1973 both δ and Π were zero. The introduction of a dividend yield δ was made by Merton (1973). The extension to include a payment flow $\Pi(t,P)$ can be seen in Dixit and Pindyck (1994).

Applying Ito's lemma on a function $Z(t) = e^{-r(t-t_0)}V(t,P)$, we get the diffusion dZ as

$$dZ = (Z_t + \kappa PZ_P + \tfrac{1}{2}\sigma^2 Z_{PP})dt + \sigma Z_P\, dv\,,$$

where $Z_t = -re^{-r(t-t_0)}V + e^{-r(t-t_0)}V_t$

$Z_P = e^{-r(t-t_0)}V_P$

$Z_{PP} = e^{-r(t-t_0)}V_{PP}$.

Substitution of the partial derivatives into the expression for dZ gives

$$dZ = e^{-r(t-t_0)}(V_t + \kappa PV_P + \tfrac{1}{2}\sigma^2 V_{PP} - rV)dt + e^{-r(t-t_0)}\sigma V_P dv\,. \tag{2.8}$$

Now, let V in equation (2.8) be a solution to the differential equation (2.6). This is perfectly in order. We only say that the differential equation of (2.6), with a variable defined as in (2.7), must have a solution. Substituting (2.6) into (2.8) gives

$$dZ = -e^{-r(t-t_0)}\Pi(t,P)dt + e^{-r(t-t_0)}\sigma V_P dv\,.$$

This expression is no formal equation, but a representation of the integral equation

$$Z(T) = Z(t_0) - \int_{t_0}^{T} e^{-r(t-t_0)}\Pi(t,P)dt + \int_{t_0}^{T} e^{-r(t-t_0)}\sigma V_P(t)dv(t)\,.$$

Taking the expected value, and noting that the expected value of a deterministic Ito-integral is zero, the expression becomes,

$$E^Q\left[Z(T) + \int_{t_0}^{T} e^{-r(t-t_0)}\Pi(t,P)dt\right] = Z(t_0)$$

$$E^Q\left[e^{-r(T-t_0)}V(T) + \int_{t_0}^{T} e^{-r(t-t_0)}\Pi(t,P)dt\right] = e^{-r(t_0-t_0)}V(t_0,P)$$

which gives the final form of the solution as

$$V(t_0,P) = e^{-r(T-t_0)}E^Q\left[V(T,P)\right] + \int_{t_0}^{T} e^{-r(t-t_0)}E^Q\left[\Pi(t,P)\right]dt\,. \tag{2.9}$$

The Feynman-Kac formula states that the value of an asset today, equals the discounted value of the expected payment at maturity, plus the discounted value of all expected payments that will be received before maturity. An amazingly simple solution to the partial differential equation earlier derived. However, note that the expectation should be computed for a price

P(t) following the diffusion process (2.7), $dP = \kappa(t,P)Pdt + \sigma(t,P)dv$. Hence the notation $E^Q[X]$.

This is the price process that pulp would follow in a so-called risk neutral world, where investors do not require compensation for systematic risk. Rewriting (2.4), the expected price increase is the difference between the risk-adjusted return and the convenience yield,

$$\frac{1}{dt}E\left[\frac{dP}{P}\right] = \mu - \delta(t,P).$$

With the convenience yield $\delta(t, P)$ unchanged and total return decreasing from the risk-adjusted return μ, to the riskless rate r when no compensation for risk is required, the drift rate in a risk neutral world must be $r-\delta(t, P)$, which is equal to $\kappa(t, P)$[9].

The essence of a real option approach is therefore to pretend that investors are indifferent to risk and calculate the value under this assumption.[10] The result will be valid even when investors are not indifferent to risk.

Option pricing and the capital asset pricing model

The assumption of risk neutrality is just a computational trick, albeit a useful one, since it can be used to discount payments that do not fit into the framework of the capital asset pricing model.

CAPM is a one-period equilibrium model and the extension to a multiperiod setting is not easily made. Fama (1977) shows that discounting the expected future payments using a single risk adjusted rate of return, requires the covariance with the market to be non-stochastic, i.e. the systematic risk of the payment is not allowed to change over time. Option pricing evades this problem through the creation of the instantaneous risk-free portfolio that can be used to replicate the payment. Even though the risk changes over time in a stochastic way, the portfolio can be maintained as risk-free by revising its composition, thus allowing valuation of the future payment.

The cost of this ability is the requirement of an underlying tradable asset, following a specified stochastic process, and the frequent updating of the portfolio. The advantage is, however, the ability to value any arbitrary contract as long as the above conditions are met. Specifically, the cash flow resulting from the simultaneous price and production size uncertainty can now be valued. When production size is altered as a response to changes in the market price, cash flow as a function of market price becomes convex. The risk will thereby change over time depending on the (stochastically changing) market price.

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[9] $r - \delta(t,P) = r - \mu + \frac{1}{dt}E\left[\frac{dP}{P}\right] = \kappa(t,P)$

[10] Real option literature is also concerned to a great extent with the optimality problem of when to invest. The NPV criteria just says if an investment is good or bad and is not concerned with optimal timing.

More generally, the risk will change for any payment not being a linear function of the underlying asset. Such a payment will henceforth be called an asymmetric payment. A terminology used by, for example, Trigeorgis and Mason (1987). In order for the CAPM to handle asymmetric payments, it would have to be updated instantaneously, which, in fact, was one of the ideas leading to the Black and Scholes differential equation, see Black (1989). In their original paper from 1973, Black and Scholes provide a derivation of their differential equation using the CAPM.

The short rate of return

In the derivation of the Feynman–Kac formula, a constant short rate of return was used. More generally, the short rate can be any deterministic function of time. This is also in accordance with the multiperiod CAPM since the discount rate used in a net present value calculation should, to quote Fama (1977), be "..known and non-stochastic, but the rates for the different periods preceding the realisation of the cash flow need not be the same,..".

If the short rate is not constant, but still deterministic, the discount factor $e^{-r(T-t_0)}$ should be substituted by $e^{-\int_{t_0}^{T} r(t)dt}$. Letting $\bar{r}$ denote the average risk-free short rate, the discount factor can be written as $e^{-\bar{r}(T-t_0)}$.

This highlights a subtle and often overlooked point. When, for practical purposes, a constant discount rate is used, it is the average short rate of return that should be used. Not the short rate presently observed in the money market.

3. Data gathering

Model used

In order to value the pulp plant, it is necessary to state the assumptions more specifically. The derivation in Section 2 was based on the very general Ito process (2.1). We now specify the price process as a geometric Brownian motion. This assumption is made partly for congruence and partly for convenience. As can be seen in Appendix 1, the net present value and the Feynman-Kac formula will give the same answer for any price process $dP = \alpha P dt + \sigma(t, P) dw$. Specifying the volatility function $\sigma(t,P)$ as $\sigma \cdot P$, where σ is a constant, we arrive at the ordinary geometric Brownian motion. Observe, though, that the closer specification of the volatility function is not needed for congruence with the net present value calculation. It is only needed for the asymmetric payments, occurring when the production rate is altered in response to changing market conditions.

Parameter estimation

In the growing literature about real options, parameter estimation is a problem that has been given surprisingly little coverage. It is unclear why. Either the problem is considered trivial or deemed as an applicational aspect rather than a theoretical problem. Whatever the reason, if the real option method is ever to gain acceptance in the business community, a reasonably simple procedure to estimate the parameters is a necessity. This is one reason why the geometric Brownian motion is suitable to start with. Here, parameter estimation is particularly straightforward.

Following Björk (1994), the geometric Brownian motion, $dP = \alpha P dt + \sigma P dw$, has the solution

$$\ln P(T) - \ln P_0 = (\alpha - \tfrac{1}{2}\sigma^2)(T - t_0) + \sigma w(T - t_0).$$

Defining $X(T)$ as the normally distributed variable

$$X(T) = \ln \frac{P(T)}{P_0} \sim N\left[\left(\alpha - \tfrac{1}{2}\sigma^2\right)(T - t_0), \sigma\sqrt{T - t_0}\right], \tag{3.1}$$

gives the discrete observations as

$$X(t_{k+1}) = \ln \frac{P(t_{k+1})}{P(t_k)} \sim N\left[\left(\alpha - \tfrac{1}{2}\sigma^2\right)\Delta t, \sigma\sqrt{\Delta t}\right].$$

Estimating parameters in the normal way, we have the mean as

$$\left(\alpha - \tfrac{1}{2}\sigma^2\right)\Delta t = \bar{x} = \frac{1}{n}\sum x_k ,$$

and the standard deviation

$$\sigma\sqrt{\Delta t} = s = \sqrt{\frac{1}{n-1}\sum\left(x_k - \bar{x}\right)^2}\ .$$

The parameter estimates are then

$$\sigma = \frac{s}{\sqrt{\Delta t}} \text{ and } \alpha = \frac{\bar{x}}{\Delta t} + \tfrac{1}{2}\sigma^2 .$$

Market data

Parameters are based on continuously compounded historical market data for the period 1980-1996. Data has been collected quarterly and are detailed in Appendix 2. All data is related to Swedish crowns, although computations in U.S. dollars would yield almost identical results.

Parameters	Comments
Risk free interest rate: $r = 6.4\%$	Measured as the historical real rate plus an expected inflation of 2%.[11]
Expected price increase in pulp: $\alpha = 1.3\%$	Measured as the historical real drift, -0.7% plus 2% expected inflation.
Standard deviation of pulp prices: $\sigma = 18.9\%$	
Beta of pulp prices: $\beta = 0.16$	The return on Affärsvärldens generalindex at the Stockholm Stock Exchange is used as a proxy for the market.
Market risk premium: $r_M = 8\%$	This is an average during the 20th century, see Ibbotson and Sinquefield.
Risk adjusted discount rate: $\mu = 7.7\%$	Through CAPM (6.4 + 0.16·8)
Rate of return shortfall: $\delta = 6.4\%$	Defined as $\mu - \alpha$.

By adopting the forecast (and objective) by the Swedish Riksbank of a future inflation rate of 2%, we are projecting a lower inflation than the one inherent in historical data. Therefore, the forecasted inflation rate is added to the historical real interest rate. When it comes to drift and diffusion of the pulp price, it is interesting to note (Appendix 2) that diffusion is unchanged (σ = 18.9%), independently of whether the pulp price is expressed as real or nominal. Furthermore, the difference in drift rate is offset by the inflation rate, so the procedure to calculate the real drift rate and add the expected future inflation can be used. This is for practical applications quite important, as ambiguities are avoided.

[11] Inflation forecast is made by the Swedish Riksbank. Since interest rates are continuously compounded, the real rate and the inflation are just added to arrive at the nominal interest rate.

There is another interesting observation in the above data. The systematic risk of pulp is very low, beta equals 0.16. Forestry companies have a much higher β, often above unity, although the major insecurity is the price of forestry products.[12] Even after adjusting for financial leverage, there is a huge gap.

One explanation may be that the stock market reacts faster than price changes. For example: If there is trustworthy information of an upcoming recession, the stock market will incorporate this information immediately. However, the pulp price will not decrease until the information has been proven true and an actual recession hits the market. In this scenario we would expect low correlation between the stock market and the pulp price, with a correspondingly low β. This is also what we observe in the above data.

The lack of correspondence between pulp β and company β presents a problem for capital budgeting. Using company β to determine the riskiness of cash flow, as is often done, will give a very different result from using pulp β. A net present value calculation can be performed using whatever β is preferred. A real option calculation, on the other hand, relies exclusively on pulp β. Thus, one could not in general expect to be able to replicate a net present value calculation in a real option framework.

Plant data

The fictious, but realistic, plant used as a case study in this paper is capable of producing 400 000 tons of pulp per annum. The pulp is of the standard kraft traded: NBSK- Northern Bleached Softwood Kraft pulp, which is a sulphate pulp that is often specified 90% dry. 90% dry means that it is storable and can be sold to papermills built separately. Otherwise it is quite common within the industry to place pulp- and papermill together. The advantage is that of less drying and transportation of the pulp. However, it also becomes more troublesome to separate the pulp process, which is the reason this study deals with a plant for market pulp only. Another reason to choose this kind of plant is that dried NBSK pulp is a standard commodity with readily accessible data and also futures markets in operation.

<u>Data</u>	<u>Comments</u>
Today's price of pulp: SEK 4500 per ton.	USD 600 $\times$ 7.50 SEK/USD = SEK 4500
Milling capacity: 400 000 tonnes.	400 000 tonnes is a reasonable size. Although there are some economies of scale in a larger mill, there will probably be logistic problems in receiving enough pulpwood.
Economic life: 30 Years.	Technically, a bit on the conservative side. However, there are not many older mills operating today.

[12] Another major source of insecurity is the dollar exchange rate. The market price of forestry products is determined in dollars. However, at least for pulp, computations in dollar and using dollar stock markets, give similar results. Hence, it does not explain the deviation.

Investment costs: SEK 4 500 million.	Wood handling 300, Digesting, Screening and Washing 700, Bleach plant 500, Recovery boiler 800, Evaporation unit 400, Recaustizing 400, Logistics 400, Drying 1 000.
Cost of pulpwood: 30% of pulp price.	Generally speaking, when the price of pulp changes, so does the price of pulpwood. 30% is the average cost.
Other variable costs: SEK 1250 per ton.	Chemicals, energy and transportation.
Maintenance: SEK 150 million the first 15 years, thereafter 250 million.	This is a simplification in order to reduce the amount of calculation needed. Presumably, maintenance costs follow a parabola. Costs are low when the machinery is new and when abandonment is close and higher in between.
Other fixed costs: SEK 300 million.	Whereof 70% are wages.
Salvage value: Zero.	Costs of site recovery and the value of being able to continue operation are minor. See Section 7, the expansion option, for a more detailed discussion.

The costs have been obtained through interviews with industry representatives and should be seen as reasonable, but not necessarily true for any specific plant. Real costs will decrease in the future due to continuous productivity gains. According to the industry representatives, pulp has shown a long-term price decline of 1 % per annum in real terms. Naturally, even the costs of production have decreased, as there otherwise would be no producers left. As mentioned in the previous section of market data, the real price of pulp has for the 1980-96 period decreased with an average of 0.7 % per annum. We will use the same assumption for costs and therefore, with 2 % inflation, let the costs increase with 1.3% over time. Costs will be discounted at the risk-free rate since they are assumed to be quite stable and not correlated with market return.

In order to judge the realism of the data, a Profit and Loss Account can be helpful. Taking the initial price of 4500 SEK per ton as given, the accounts for the first year will look like:

	Year 1 (MSEK)
Pulp sales (400 000 tonnes)	0.4·4500
- Pulpwood cost (30% of sales)	- 0.3·0.4·4500
- Other variable costs	- 0.4·1250
- Maintenance	- 150
- Other fixed costs	- 300
- Depreciation (straight line, 30 years.)	- 150
	160

A small accounting profit can be expected for the first year. Also for subsequent years, a small profit can be expected. How small depends on the amount of maintenance needed and the depreciation method used. What the Profit and Loss Account fails to encompass, however, is the time value of money and the immense uncertainty of the pulp price. Let us therefore continue with a net present value calculation.

4. A fixed production policy

The net present value

(All results are in MSEK)

$$PV(\textit{ pulp sales})^{13} = \int_0^{30} e^{-0.077t}\left[0.4 \cdot 4500e^{0.013t}\right]dt = 24002$$

$$PV(\textit{ pulpwood cost})^{14} = -\int_0^{30} e^{-0.077t}\left[0.3 \cdot 0.4 \cdot 4500e^{0.013t}\right]dt = -7201$$

$$PV(\textit{ other variable costs})^{15} = -\int_0^{30} e^{-0.064t}\left[0.4 \cdot 1250e^{0.013t}\right]dt = -7681$$

$$PV(\textit{maintenance})^{16} = -\int_0^{15} e^{-0.064t}\left[150\, e^{0.013t}\right]dt - \int_{15}^{30} e^{-0.064t}\left[250\, e^{0.013t}\right]dt = -2792$$

$$PV(\textit{other fixed costs})^{17} = -\int_0^{30} e^{-0.064t}\left[300\, e^{0.013t}\right]dt = -4609$$

Added together, the present value of operating this plant is SEK 1719 million. Thus, operation of an existing plant is profitable. However, any new investment is out of question, since the owners would then have to pay the investment costs of SEK 4500 million as well.

The net present value is SEK – 2781 million.

The real option technique

The net present value can also be obtained through the Feynman-Kac formula (2.9). Using this option technique, all values should be calculated as in a so-called risk neutral world. All costs (except pulpwood) are already discounted at the risk free rate, so there is no need to repeat the calculations here. Instead we demonstrate the technique by calculating the value of pulp sales.

Using the definition of expected value,

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[13] $4500e^{0.013t}$ is the expected price and 0.4 represent the 400 000 tonnes of pulp produced each year, thereby giving the result in MSEK. Pulp sales and pulpwood costs are discounted at the risk-adjusted discount rate appropriate to the risk of pulp. All other items are discounted at the risk free rate, as these costs are assumed uncorrelated with market return.

[14] Pulpwood cost is approximately 30% of pulp price.

[15] 400 000 tonnes of pulp per year, times a variable cost of $1250e^{0.013t}$ per ton.

[16] Maintenance is 150 MSEK per year, for the first 15 years and thereafter 250 MSEK per year.

[17] Other fixed costs are 300 MSEK per year.

$$E^Q[pulp\ sales] = E^Q[0.4P(t)] = \int_{-\infty}^{\infty} (0.4P(t)) \cdot \varphi(p)dp .$$

The price variable, $P(t)$, is here lognormally distributed. In order to work with the more familiar normal distribution, we use equation (3.1) to make the variable transformation

$P(t) = P_0 e^{X(t)}$, where $X(t) \sim N\left[\left(r - \delta - \tfrac{1}{2}\sigma^2\right)t , \sigma\sqrt{t}\right]$.

Denoting $X \sim N[a(t), b(t)]$, the probability density function $\varphi(x)$ is

$$\varphi(x) = \frac{1}{b(t)\sqrt{2\pi}} e^{-\frac{(x-a(t))^2}{2b^2(t)}} ,$$

and the numerical values of $a(t)$ and $b(t)$ are

$$a(t) = \left(r - \delta - \tfrac{1}{2}\sigma^2\right)t = -0.01786t$$
$$b(t) = \sigma\sqrt{t} = 0.189\sqrt{t}$$

Using an interval of ten standard deviations, in order to avoid any round off errors, X must vary from $a(t) - 5b(t)$ to $a(t) + 5b(t)$, and the expected sales become

$$E^Q[pulp\ sales] = \int_{a(t)-5b(t)}^{a(t)+5b(t)} \left(0.4 \cdot 4500e^x\right) \cdot \varphi(x)dx .$$

Discounting using the risk-free interest rate gives today's value as

$$V_0\,(\,pulp\ sales) = \int_0^{30} e^{-0.064t} \int_{a(t)-5b(t)}^{a(t)+5b(t)} \left(0.4 \cdot 4500e^x\right) \cdot \varphi(x)\, dxdt = 24002 .$$

This is the same value as was achieved by the previous present value calculation. The motive for repeating it is to show that there is no difference in assumptions so far and later differences in firm value are fully due to changes in the production rate. With all preliminaries behind us, it is now time to model such a situation.

5. Modelling a variable production rate

The textbook rule of operation: Maximise the contribution to profit by utilising the plant to capacity as long as price exceeds variable cost, describes an ideal situation. It hinges upon many assumptions of which the most important are:

- No costs of stopping or starting production.
- There will be no market consequences if production is halted.
- Full competition is prevalent and no actor can affect the market price.
- True costs are known.

Since these assumptions are quite restrictive, the challenge facing the production manager is more complex than the simple textbook rule suggests. Of particular interest to this study is the willingness to decrease production in order to reverse a price decline. The company thereby believes that it has some discretion over market price development[18] or that other manufacturers will follow suit and decrease production.

This study is not trying to question the rationality in this behaviour, nor is the intent to find the optimal production policy. The purpose is to deduct whether it is necessary to detail different production policies when performing (advanced) capital budgeting.

It has been quite difficult to obtain a realistic production policy, even though industry statistics give a clear connection between market price of pulp and the production rate. Managers assess so much more than just the current market price. When deciding what production rate to choose, managers also consider market trends, as well as storage levels of pulp, both in the market[19] and in their own warehouses.

In spite of all the difficulties associated with, a priori, specifying a production policy, the policy here specified is not unreasonable. It should, without any pretence of being optimal or empirically correct, give an appreciation of the magnitude of change in plant value that a variable production rate accounts for. We specify the production policy as follows:

Normally the plant operates at maximum capacity and it so continues as long as price stays above SEK 3500. The profit and loss account will show red figures even above this price, but due to competition in the marketplace and the contribution to profit, nothing will happen before the price decreases to SEK 3500. Then the company will react, trying to push the price upwards by cutting production. In the range SEK 3500-2600, production decreases linearly from 100% to 70%. If price is less than SEK 2600 the production is altogether shut down and also maintenance is stopped. This is approximately equivalent to saying that all work ceases when price is less than variable costs.[20]

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[18] From a modelling point of view, we implicitly assume that the company does not have "too much" discretion over price, as the price process then would be endogenous, making valuation difficult.

[19] Market storage levels are measured by the so called NORSCAN level. NORSCAN stands for North America and Scandinavia, and measures the amount of pulp warehoused in these regions.

[20] Only approximately equivalent, as pulpwood cost changes with the level of pulp price and there are semi-fixed maintenance costs to consider. Although the price of pulpwood mirrors that of pulp for reasonable price levels, it is unlikely that this will be the case if the pulp price is very low. Wood can be used for other purposes than pulp

Price > 3500

An expected increase in price (and costs) of 1.3% per annum, requires the lower bound to be specified as $P(t) \geq 3500\, e^{0.013t}$. Using *X* as the stochastic variable (since it is normally distributed) with $P(t) = 4500e^{X}$, the lower boundary for *X* becomes:

$$X \geq \ln\frac{3500}{4500} + 0.013t\,.$$

The upper boundary of a normally distributed variable is, of course, infinity, but for computationally convenience we confine the boundary to five standard deviations.

$$V_0\textit{ (pulp sales)} = \int_0^{30} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (0.4\cdot 4500e^{x})\cdot\varphi(x)dxdt = 17629$$

$$V_0\textit{ (pulpwood cost)} = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (0.3\cdot 0.4\cdot 4500e^{x})\cdot\varphi(x)dxdt = -5289$$

$$V_0\textit{ (other variable costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (0.4\cdot 1250e^{0.013t})\cdot\varphi(x)dxdt = -3918$$

V₀ (maintenance) =

$$-\int_0^{15} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (150e^{0.013t})\cdot\varphi(x)dxdt - \int_{15}^{30} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (250e^{0.013t})\cdot\varphi(x)dxdt = -1333$$

$$V_0\textit{ (other fixed costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{3500}{4500}+0.013t}^{a(t)+5b(t)} (300e^{0.013t})\cdot\varphi(x)dxdt = -2351$$

production. The figure SEK 2600 is obtained by adding the variable costs of SEK 1250, to the initial cost of pulpwood, which for a pulp price of SEK 4500, is 30% · 4500 = 1350.

2600 < Price < 3500

As the price decreases, so does the production rate. Utilisation decreases linearly from 100% for a price of 3500 to only 70% when the price is 2600. Not only production and variable costs are reduced within this price range. Also maintenance is cut. It is possible to cut down on maintenance since maximum output is not an issue. Even if the plant is out of operation for a while, this is no major issue since it is possible to catch up on production later.

Denote the level of utilisation with *f(P).* In nominal terms, utilisation changes linearly from 70% when the price equals $2600e^{0.013t}$ to 100% when the price is $3500e^{0.013t}$. Utilisation as a function of price will then be the straight line $f(P) = \frac{e^{-0.013t}}{3000}P - 0.167$. Expressed in the variable X instead, the utilisation function becomes $f(X) = \frac{e^{-0.013t}}{3000} \cdot 4500e^{X} - 0.167$.

The integration limits for the stochastic variable X, is
$\ln\frac{2600}{4500} + 0.013t \le X \le \ln\frac{3500}{4500} + 0.013t$.

$$V_0\,(\textit{pulp sales}) = \int_0^{30} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} f(x)\cdot\left(0.4\cdot 4500e^{x}\right)\cdot\varphi(x)dxdt = 2750$$

$$V_0\,(\textit{pulpwood cost}) = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} f(x)\cdot\left(0.3\cdot 0.4\cdot 4500e^{x}\right)\cdot\varphi(x)dxdt = -825$$

$$V_0\,(\textit{other variable costs}) = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} f(x)\cdot\left(0.4\cdot 1250e^{0.013t}\right)\cdot\varphi(x)dxdt = -1116$$

V_0 *(maintenance)* =

$$-\int_0^{15} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} f(x)\cdot\left(150e^{0.013t}\right)\cdot\varphi(x)dxdt - \int_{15}^{30} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} f(x)\cdot\left(250e^{0.013t}\right)\cdot\varphi(x)dxdt$$

= - 388

$$V_0\,(\textit{other fixed costs}) = -\int_0^{30} e^{-0.064t} \int_{\ln\frac{2600}{4500}+0.013t}^{\ln\frac{3500}{4500}+0.013t} \left(300e^{0.013t}\right)\cdot\varphi(x)dxdt = -786$$

Price < SEK 2600

Changing the price variable gives the upper integration limit as $X \leq \ln\frac{2600}{4500} + 0.013t$.

The lower integration limit of minus infinity is for computational convenience confined to five standard deviations, $a(t) - 5b(t)$. Only fixed costs are present when the price is less than 2600 Swedish crowns.

$$V_0 \textit{ (other fixed costs)} = -\int_0^{30} e^{-0.064t} \int_{a(t)-5b(t)}^{\ln\frac{2600}{4500}+0.013t} \left(300e^{0.013t}\right)\cdot \varphi(x)dxdt = -1472$$

Totally

Added together, V_0 *(future cash flow)* = 2901 MSEK. In the case of a present value calculation, *PV* = 1719 SEK. As the investment cost of 4500 SEK has to be subtracted in both cases, the investment is not worthwhile in either case. The difference between the two valuations, roughly 1200 MSEK, is the additional value of the specified production policy. This difference will increase the more volatile the market is, as the probability of a low price thereby increases, making loss cutting policies all the more important. On the other hand, a higher drift rate must reduce the difference, as a low price then is less probable.[21] Hence, the differences are parameter dependent and should also be dependent upon the price process specified.

There are two counteracting effects accounting for the difference in value. First and foremost the loss cutting procedure of closing the plant whenever price is less than variable costs. This increases the value by some 1250 MSEK.[22] Secondly, reducing production although price is above variable costs will result in a loss of contribution to profit by some 50 MSEK. Thus, from a capital budgeting point of view, the eagerness to restrain a price decline by decreasing production seems of less interest. The important thing is to stop production whenever price is less than variable costs.

[21] For example, allow the plant to break even under the variable production scheme. This can be achieved increasing σ from 0.189 to 0.33, holding all other variables constant. The difference between the two policies is now 2800 MSEK. If we instead adjust the drift rate α, from 1.3 to 2.3%, the plant will also break even under the variable production scheme, but the difference is now only 500 MSEK.

[22] Changing the integration limits so that full production is sustained for a price exceeding SEK 2600 per ton separates this effect.

Observe, however, that production is only stopped when the price declines below SEK 2600 per ton. As the price has never been this low, historically, it is easy to suspect the price process of assigning relatively high probabilities to rather unlikely outcomes. The purpose of the next section is therefore to study the price process in more detail.

6. Modelling the pulp price as mean reverting

The geometric Brownian motion assumed so far, is not uncontested as the model of how prices behave. It is definitely the most natural candidate, given its many advantages: The geometric Brownian motion is relatively easy to understand and has an explicit analytical solution. It is also congruent with net present value calculations and parameter estimation is fairly simple. Furthermore, as a model of stock price behaviour, a constant relative drift rate plus the normally distributed noise, conform nicely to how we (perhaps naively) would expect stock prices to behave.[23]

However, for the movements of commodity prices, there are some compelling arguments why the behaviour should not be modelled in this way. If the pulp price is exceptionally high, this will presumably attract new producers trying to profit from the situation, with a price decline as a result of the increased competition. Taking the other extreme, when price is below marginal cost, some producers will be forced out of the market, leaving the others struggling to increase the price.

Undoubtedly, a "truer" model of price movements should capture this equilibrium characteristic, called mean reversion. However, the geometric Brownian motion is not easy to disclaim empirically. Pindyck and Rubinfeld (1991) performs in chapter 15 a unit root test, where they are only able to reject a random walk of copper and crude oil prices when more than 100 years of data are used. Even so, they fail to reject a random walk in the price of lumber. Other authors are not so sure. Schwartz (1997) found strong mean reversion in futures prices of copper and oil, with significant coefficients.[24]

It is of interest to calculate the value of the pulp plant under the assumption that the price is mean reverting. First and foremost because we earlier saw that a large part of the different results between a fixed and a variable production rate came from price levels that might (arguably) be unrealistically low. It is also important because careful analyses, allowing for mean reversion and other characteristics, should always be undertaken before committing capital to a major investment.

The simplest and most well known mean reverting process is the Ornstein-Uhlenbeck process, popularised by Vasicek (1977) as a model of how interest rates behave. It will also serve as a starting point for describing pulp price behaviour. The infinitesimal characteristic reveals the basic property of the process,

$$dP = \eta\left(\bar{P} - P\right)dt + \sigma dw. \tag{6.1}$$

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[23] A constant relative drift rate represents the fact that some of the operating profits are reinvested and can be expected to earn the same return as existing funds. We could thus expect the stock price to appreciate over time, but have to add a diffusion term accounting for new information. This diffusion will be normally distributed (the central limit theorem) if it is caused by many independent pieces of news.

[24] It is interesting to note that Schwartz fails to verify mean reversion in the price of gold futures. The coefficients are not significantly different from zero. Gold is often considered an investment asset rather than a commodity, and we would therefore be more tempted to model the price process as a geometric Brownian motion.

If $\bar{P} < P$ this process exhibits a negative drift and when $\bar{P} > P$ the drift is positive. Hence, the (real) price of pulp is driven back to its long-term average $\bar{P}$ with a speed of reversion η.

One of the characteristics of the Ornstein-Uhlenbeck process is that the price is normally distributed. Although this is computationally convenient, it lacks economic appeal in that negative prices thus are allowed. This is not so for the previously used geometric Brownian motion. There, negative prices are disallowed because it is the logarithm of the pulp price that is normally distributed. The pulp price is thereby, by definition, lognormally distributed.

Using the same logic, it is natural to suggest that the pulp price could be modelled by letting the logarithm of the price follow an Ornstein-Uhlenbeck process. This is also the approach taken by Schwartz (1997). Ekvall, Jennergren, Näslund (1995) make use of the same process for modelling the spot exchange rate and uses it to value currency options.

However, in one important respect, commodity prices are different from the exchange rates underlying currency options. Commodity prices are subject to inflation. Whereas it is difficult to see why an exchange rate should exhibit any long-term drift, commodity prices do increase over time. Also, technological improvements may well lead to a drift rate separated from the rate of inflation, precluding a use of real discount rates.[25]

Therefore, this paper proposes a model where the nominal price is mean reverting. Instead of assuming the equilibrium price $\bar{P}$ to be a constant, we allow it be time dependent with $\bar{P}(t) = \bar{P}_0 e^{\omega t}$.

Taking the logarithm of the equilibrium price, we get

$$\ln \bar{P}(t) = \gamma + \omega t \quad \text{with} \quad \gamma = \ln \bar{P}_0 . \tag{6.2}$$

The proposed model will be

$$dP = \eta(\gamma + \omega t - \ln P)P dt + \sigma P dw . \tag{6.3}$$

This model will allow both mean reversion and an equilibrium price that increases over time, as well as disallow negative prices. By defining $X(t) = \ln P(t)$ as is done in Appendix 3, the process reduces to

$$dX = \eta(\gamma' + \omega t - X) dt + \sigma dw \quad \text{where} \quad \gamma' = \gamma - \frac{\sigma^2}{2\eta} . \tag{6.4}$$

Without drift, $\omega = 0$, the process (6.4) becomes an Ornstein-Uhlenbeck process of the type (6.1). The derivation in Appendix 3, shows that $X(T)$ is normally distributed with mean

[25] The use of real discount rates may also interfere with the ability to replicate the cash flow of the derivative asset. Cash flows are almost always nominal and trying to lock in an arbitrage profit from a mispricing of the real cash flow can sometimes be hard work.

$$E[X(T)] = \left(\gamma' - \frac{\omega}{\eta}\right)\left(1 - e^{-\eta(T-t_0)}\right) + \omega\left(T - t_0\, e^{-\eta(T-t_0)}\right) + X(t_0)e^{-\eta(T-t_0)} \tag{6.5}$$

and variance

$$Var[X(T)] = \int_{t_0}^{T} \sigma^2 e^{-2\eta(T-t)} dt = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta(T-t_0)}\right). \tag{6.6}$$

Parameter estimation

There is no generally agreed way of parameter estimation for mean reverting processes and Appendix 4 gives comments on three different methods. The one used, inspired by Harvey (1989) pp. 481-82, is arguably the most straightforward and is congruent with the way the parameters of the geometric Brownian motion were estimated.

For both types of stochastic processes, the logarithm of the price is normally distributed. In the case of the geometric Brownian motion, also the return is normally distributed, making parameter estimation straightforward. The additional *t* term in the mean reverting process complicates matters, but since

$$X(t_{k+1}) = \left(\gamma' - \frac{\omega}{\eta}\right)\left(1 - e^{-\eta\Delta t}\right) + \omega\,\Delta t + \omega\left(1 - e^{-\eta\Delta t}\right)t_k + e^{-\eta\Delta t} X(t_k) + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}}\,\varepsilon\,, \tag{6.7}$$

where $\varepsilon \sim N(0,1)$, it is nevertheless possible to run the regression

$$X(t_{k+1}) = c_0 + c_1\, t_k + c_2\, X(t_k) + s\,\varepsilon\,, \tag{6.8}$$

to estimate the parameters. The details are in Appendix 4, but the result follows immediately from the expressions of the mean and variance, equation (6.5) and (6.6).

Although the regression (6.8) looks straightforward, it nevertheless poses an inherent problem, that of multicollinearity. As prices tend to increase over time, t_k and $X(t_k)$ will be correlated, affecting parameter estimations as a result. Harvey (1989) avoids this problem as he deals with a standard Ornstein-Uhlenbeck process, where the equilibrium price is not time dependent.

If price was constant in real terms, working with real price data would be a solution as time and price then would be uncorrelated.[26] However, the pulp price has decreased in real terms and the correlation between time and price is around 0.6 for both real and nominal data.

One typical remedy to the multicollinearity problem is to delete some of the collinear variables, thereby improving the precision of the remaining regression coefficients. See, for example, Canavos, 1984, pp. 485.

[26] If t_k and $X(t_k)$ were orthogonal, in the sense that Corr[t_k , $X(t_k)$] = 0, the regression coefficients would be unaffected by the number of dependent variables used.

Following this tradition, we delete time and use the logarithm of the real price as the only dependent variable in the regression. Formally, the regression run is the AR(1) process

$$X(t_{k+1}) = c_0 + c_2\, X(t_k) + s\,\varepsilon \tag{6.9}$$

with

$$c_0 = \gamma'\left(1 - e^{-\eta\,\Delta t}\right)$$
$$c_2 = e^{-\eta\,\Delta t},$$

since ω in equation (6.7) is zero. As given by Appendix 4, the parameter estimations will be:

$$\eta = 0.25$$
$$\sigma = 0.19$$
$$\gamma = 8.56$$

The next step is to give a separate estimation of ω, the expected increase in the equilibrium price, as it was left out of the regression. It seems reasonable to give ω the same numerical value (1.3%) as the drift rate of the geometric Brownian motion. Choosing another value, and thus different values for the drift of the geometric Brownian motion and the equilibrium drift of the mean reverting process, makes comparisons difficult.

The main difference between the two processes is clearly shown in Diagram 6.1, where 95 percentage confidence intervals are depicted. Whereas the mean reverting process is confined to a price range which most people would conceive as reasonable, the geometric Brownian motion can sometimes wander far off and away from any economic reality. This explains why the loss cutting procedure in Section 5 had such an influence on plant value. It is possible that the price will move below variable costs and stay there for extended periods of time.

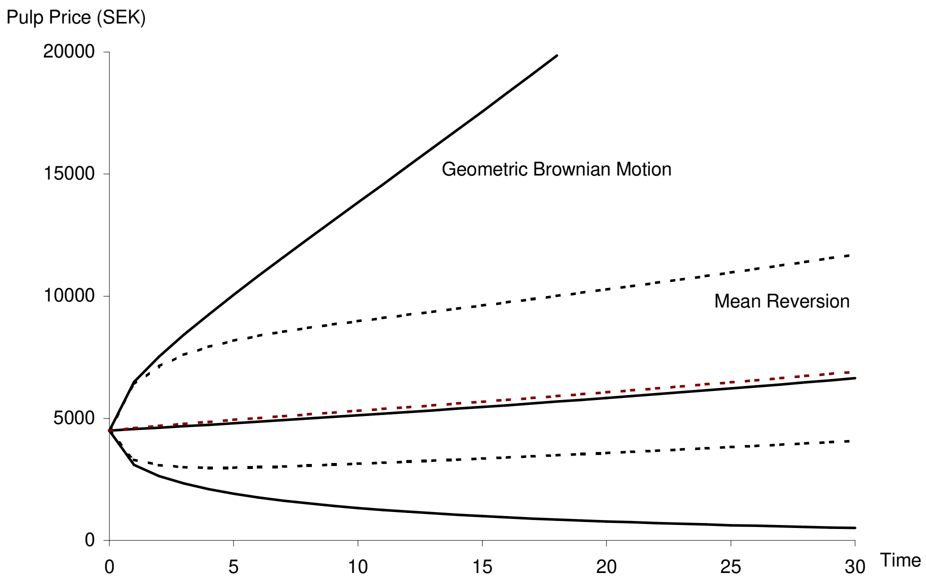

Diagram 6.1, 95 percentage confidence intervals and expected values for the development of pulp prices.

Diagram 6.1 also points at the importance of choosing $\omega = 1.3\%$, as the expected value of the two processes thereby will be approximately the same. This congruence could not have been obtained with a less advanced mean reverting process. Such processes treat the real price as constant, whereas the pulp price has decreased in real terms.

As for the geometric Brownian motion, managers may question, and rightly so, why they should trust a valuation based on a price process that does not say anything about the price level 15 years from now, let alone 30 years? The proposed mean reverting process has a large advantage in this respect. As usual, there are some drawbacks. Parameter estimation is much more difficult, due to the time dependence, and therefore more uncertain. The mathematics is also more complicated, increasing the risk of errors and hampering understanding.

The risk-neutral process

Following the derivation in Section 2., we want the Black and Scholes differential equation (2.5) to be satisfied.

$$V_t(t,P) + (r - \mu + \frac{1}{dt}E\left[\frac{dP}{P}\right])PV_P(t,P) + \tfrac{1}{2}\sigma^2 P^2 V_{PP}(t,P) + \Pi(t,P) - rV(t,P) = 0\,.$$

For the mean reverting process (6.3), the drift is

$$\frac{1}{dt}E\left[\frac{dP}{P}\right] = \eta\,(\gamma + \omega t - \ln P),$$

and the differential equation can therefore be written as

$$V_t + [r - \mu + \eta\,(\gamma + \omega t - \ln P)]PV_P + \tfrac{1}{2}\sigma^2 P^2 V_{PP} + \Pi - rV = 0\,.$$

Substituting $\psi = \gamma + \dfrac{r-\mu}{\eta}$, in order to simplify the notation, gives the Black and Scholes differential equation as

$$V_t + \eta\,(\psi + \omega t - \ln P)PV_P + \tfrac{1}{2}\sigma^2 P^2 V_{PP} + \Pi - rV = 0\,.$$

Thus, for the pricing of derivatives, the risk-neutral process

$$dP = \eta(\psi + \omega t - \ln P)Pdt + \sigma\, Pdv\,, \tag{6.10}$$

should be assumed and under this assumption the Feynman-Kac formula (2.7) will still apply.

Rather than working with the process (6.10), we make the substitution $\ln P(t) = X(t)$, as $X(t)$ then will be a normally distributed variable, dictated by the process

$$dX = \eta\left(\psi' + \omega t - X\right)dt + \sigma dv \quad \text{where} \quad \psi' = \psi - \frac{\sigma^2}{2\eta}.$$

Numerically, the parameter ψ' has the value

$$\psi' = \psi - \frac{\sigma^2}{2\eta} = \gamma + \frac{r-\mu}{\eta} - \frac{\sigma^2}{2\eta} = 8.56 + \frac{0.064 - 0.077}{0.25} - \frac{0.19^2}{2 \cdot 0.25} = 8.44\,.$$

It is now possible to use the notation

$$P(t) = e^{X(t)},\ X(t) \sim N\left[a(t), b(t)\right], \tag{6.11}$$

where

$$a(t) = \left(\ln P_0\right)e^{-\eta t} + \omega t + \left(\psi' - \frac{\omega}{\eta}\right)\left(1 - e^{-\eta t}\right) = \left(\ln 4500\right)e^{-0.25t} + 0.013t + \left(8.44 - \frac{0.013}{0.25}\right)\left(1 - e^{-0.25t}\right)$$

$$b(t) = \sigma\left(\frac{1 - e^{-2\eta t}}{2\eta}\right)^{1/2} = 0.19\left(\frac{1 - e^{-2 \cdot 0.25t}}{2 \cdot 0.25}\right)^{1/2}.$$

The risk neutral process of dX and the parameters $a(t)$ and $b(t)$, follow immediately from their real world counterparts, (6.4)-(6.6). Note that the mean reverting process cannot be written in the form $P(t) = P_0 e^{X(t)}$. Contrary to the geometric Brownian motion, the starting value P_0 cannot be separated from the parameter $a(t)$. Calculations will therefore look a bit different, depending on the stochastic process used.

Plant valuation

As in Section 4., we calculate today's value of future sales as

$$V_0\,(pulp\ sales) = \int_0^{30} e^{-0.064t} E^Q[pulp\ sales]dt = \int_0^{30} e^{-0.064t} E^Q[0.4 \cdot P(t)]dt\,.$$

The only difference being the mean reverting process specified in this section. The expected value of $P(t)$, parameterised as in (6.11), is easily obtained through completion by squares and the result is

$$E^Q\left[P(t)\right] = \int_{-\infty}^{\infty} \frac{e^x}{b(t)\sqrt{2\pi}}\, e^{-\frac{(x-a(t))^2}{2b^2(t)}}\, dx = e^{a(t) + \frac{1}{2}b^2(t)}.$$

Thus, today's value of future sales is

$$V_0\,(pulp\ sales) = \int_0^{30} e^{-0.064t}\left[0.4e^{a(t)+\frac{1}{2}b^2(t)}\right]dt = 28018\,.$$

With pulpwood costs as 30% of sales and the rest of the costs unaffected by the pulp price (and therefore the same as in Section 4.), V_0 *(future cash flow)* = 4531 MSEK. Taking the investment cost of 4500 MSEK into consideration, the value added is 31 million Swedish crowns. Nearly three billion higher than the net present value based on a geometric Brownian motion. The result is, however, very sensitive to changes in pulp price parameters and techniques of parameter estimation. It cannot alone be used as evidence that the pulp plant investment is actually worthwhile.

Using the variable production policy, described in Section 5, V_0 *(future cash flow)* = 4499 MSEK. Calculations are shown in Appendix 5. The difference in value between the two production policies, 32 MSEK, can be divided into two parts. The ability to stop production whenever price is below variable costs increases the value with 9 MSEK. The policy to reduce production, in order to reverse a price decline, decreases the value by 41 MSEK.

That the difference in value between the two production policies is smaller than the 1200 MSEK encountered in sections 4 and 5 is not surprising. The band of probable pulp prices is narrower for the mean reverting process. The production policy therefore becomes less important since the probability of low pulp prices is small. Even so, it is surprising that the difference in value must be considered as negligible. Only for the geometric Brownian motion, there seems to be a benefit of modelling a variable production rate. However, the geometric Brownian motion is not suitable for long-lived projects, and should not be used anyway, as the range of probable prices grows unbounded over time.

It is also interesting to note that the possibility of really high prices, under the geometric Brownian motion, apparently is not enough to make up for the unfavourable outcomes. Value is less than for the mean reversion assumption. This is as could be expected. Price shocks that occur will have no permanent effect in a mean reversion model. Eventually, the price will stabilise around the long run equilibrium and the variance is therefore bounded as is revealed by equation (6.6).

This is not compatible with the use of a single risk adjusted discount rate. Let $e^{-(r+k)t}$ be the discount factor, with $r \cdot t$ as the risk free part and $k \cdot t$ as the risk-compensating part. The factor $k \cdot t$ depends on the systematic risk and not the total risk. However, if the total risk is bounded, as it is in a mean reversion case, we cannot allow the compensation for systematic risk to grow unbounded. k must therefore decrease over time, with a higher discounted value as a result. Using the terminology of Laughton and Jacoby (1995), this is the risk-discounting effect of mean reversion.

7. Other options present in the pulp industry

The possibility to alter the scale of production in response to changing market conditions is just one option suggested in the real options literature. There are many more. Corporate finance textbooks usually detail three categories: Abandonment options, expansion options and timing options. [27]

The abandonment option

The abandonment option comes in many disguises. From an outright abandonment of the investment, to the slight scaling of production that has been specified in this paper. There are stages in between, like a temporary shutdown.

In the pulp industry, a temporary shutdown is a viable alternative to decrease the speed of production. A shutdown has the advantage of not lowering the process yield (which will be the effect if the plant is not running to full capacity), but instead there are costs of restart. Unfortunately, shutdowns are not possible during wintertime; the plant would freeze. At least in Canada and Scandinavia where the majority of the world's softwood pulp is produced. During the summer, any extended shutdown will instead result in bacterial problems in the wet pulp. Longer shutdowns are rare in the pulp industry, since conservation of the processes, e.g. the recovery boiler, is quite expensive.

It is not obvious that decreasing the speed of production, as is done in this paper, is preferable to a temporary shutdown. The preferences will differ from company to company. From a capital budgeting viewpoint, however, a lower capacity usage is much easier to model and that is the reason why it has been used in this paper. To build a model where shutdowns are incorporated is certainly possible, although much more complicated, since not only the present price of pulp matters, but also the status of the plant (in operation, stopped, shut down etc.), needs to be considered. It is however, doubtful that such a model would bring any major improvement in accuracy compared with the calculations here shown, as the difference is largely technical.

The expansion option

The option to expand has to do with the competitive edge that operating the plant today renders you in the future. By producing today, you may be in a better position to make future investments. The establishment of market relations, technical know-how, established organisational procedures etc., sometimes summarise to a valuable option. It is an option because the decision to invest has not yet been made, and the opportunity to do so may to some extent be unknown.

What is the value of the expansion option for the pulp industry? Judging from the stock market not much. In stock valuation, it is common to talk about the Net Present Value of

[27] See, for example, Ross, Westerfield, Jaffe, (1996), chapters 8 and 21. For an extensive survey, Trigeorgis (1996) chapter 1 is recommended.

Growth Opportunities, as the difference between today's free cash flow generating capacity and the stock price. The NPVGO is the option to expand, branded under another name.

Even without any formal analysis, one can conclude that the low Price-Earnings ratios for most forestry companies in Sweden, is indicative of very few growth opportunities. This conclusion is further supported by the fact that the market values have been less than book values for extended periods of time during the 1990:s.[28]

In addition, the industry representatives I have interviewed, do not believe that there are any growth options attached to the production of the standard commodity considered here - Northern Bleached Softwood Kraft pulp. During the life of the mill, it is possible to trim production by 10%. However, it is not possible to increase production from 400 000 tonnes to, say, 600 000 tonnes. In this case it would be cheaper to build a new plant.

After 30 years, when the exemplified pulp mill has outlived its economic life, it is time to decide if a new investment should be made. Are we, thanks to the previous investment, in a better position than would otherwise have been the case? If so, this is an option because the investment is 30 years ahead and we have no commitment to undertake it. The option is also a pulp derivative. The future investment will be carried through only if the pulp price is high enough to motivate it.

If the decision to build a new mill is made, it will probably be situated next to the old one. Thereby a smooth transition from the old facility to the new can be accomplished and, perhaps, a few stages from the old mill can be used in the new one. Also, already trained personnel will be employed, reducing the time needed to run-in the new mill.

What is the value of this option? Suppose that some of the logistics facilities can be used: pulpwood unloading, water supply and the purification plant, etc. The value of it would be 400 million SEK. The running-in time is also reduced and an extra 100 000 tonnes can be produced during the first year, giving a contribution to profit of 190 million SEK.[29] Under the assumption that an investment only will occur if the pulp price (in year 30) exceeds SEK 4000 per ton, the value is:[30]

$$V_0(\textit{expansion option, Brownian motion}) = e^{-0.064\cdot t} \int_{\ln\frac{4000}{4500}+0.013t}^{a(t)+5b(t)} 590e^{0.013t} \cdot \varphi(x)dx = 28 \text{ MSEK}$$

$$V_0(\textit{expansion option, mean reversion}) = e^{-0.064\cdot t} \int_{\ln(4000)+0.013t}^{a(t)+5b(t)} 590e^{0.013t} \cdot \varphi(x)dx = 81 \text{ MSEK}$$

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[28] A lower market- than book value can also be interpreted as showing that an investment has negative value. The future profits (i.e. market value) are not enough to cover the (previous) investment outlays (book value). Formally, this is the concept of Tobin's Q. See, for example, Dixit and Pindyck (1994).

[29] 0.1 million tonnes · [4500 (pulp price) – 0.3·4500 (pulpwood) – 1250 (variable cost)] = 190 million

[30] For the geometric Brownian motion, a, b and φ refers to the definitions in Section 4. For the mean reverting process, the definitions are in Section 6. t is fixed as 30 years.

It is notable that the options value under the assumption of the geometric Brownian motion is less than half that of the mean reverting process. The price process specification is of paramount importance also for the expansion option.

It may be argued that the above conditions are overly simplified. The decision to invest is likely to hinge not only on the pulp price in the year 30, but at the average price for the years preceding the investment decision. It seems reasonably to assume that managers will consider the history of prices, say the last ten years, before committing money to a new pulp-mill investment. The expansion option is thereby path dependent, i.e. it does not depend on the price of pulp at a specific date, but on the whole trajectory of prices.[31] Such options are in the finance literature known as Asian options and valuation is, generally, quite hard work.

However, when the average is measured as the geometric average, analytical solutions are possible, as the geometric average of correlated lognormal variables also is lognormal.[32] In the case of the geometric Brownian motion, the distribution of the continuously observed geometric average is derived in Appendix 6, and letting the averaging period be between years 20-30, the value of the expansion option is,

V_0*(expansion option, Brownian motion)* = 32 MSEK.

Modelling path dependence only makes a negligible difference in value. As was the case with the variable utilisation of capacity, the exact formulation of the decision rule is subordinate to the price process assumed.

Apart form the option value of continued operation there is also the cost of disassembling the old plant. Today, such a disassembling would amount to around 500 million SEK, much of this due to asbestos decontamination. Even though asbestos is not used as a building material today and hence will be no problem in 30 years time, there is nothing indicating that disassembling will be cheaper in the future. On the contrary, thanks to the stricter environmental laws, a site recovery cost of 500 million SEK seems reasonable even in the future. The cost of site recovery is not dependent upon the price of pulp. Therefore, an ordinary present value calculation can be used, giving

$$PV(\text{site recovery}) = -e^{-0.064 \cdot t} \cdot 500 e^{0.013 \cdot t} = -108 \text{ MSEK}.$$

As the expansion option and the cost of site recovery are minor, the salvage value was for expositional clarity set to zero in Section 3.

The timing option

[31] Of course, one could certainly argue that any future decision, dependent on pulp price, is also path dependent. However, whilst a decision to temporary cut production is, at most, dependent upon the pulp prices for the last couple of months; a major investment decision is presumably hinging on the pulp price for the last couple of years.

[32] The geometric average is the product of all observations raised to the power of 1/n. The arithmetic average is the sum of all observations multiplied by 1/n.

Even if the present value of future payments exceeds the investment cost, it is not obvious that the investment should be made. Sometimes it is preferable to wait until a later date and the argument is as follows:

If you have an exclusive right to make an investment, for example, a concession to an oil field in the North Sea, the value of the concession will rise if the price of oil goes up. It is therefore not certain that extraction should commence immediately, even if the cash flow generated exceeds the investment cost. What makes the extraction profitable may very well be an anticipated increase in the price of crude oil. An increase in the price of crude oil will also affect the value of the concession positively. It may well be that the increase in the value of the concession makes it more profitable to keep it intact than to invest in an oilrig.

Note that this argument hinges on the existence of some sort of exclusive right. It is more profitable to keep the right to extract oil in the North See intact, than to pay the investment cost and actually start drilling, because the concession can be sold and give a higher return than the actual investment in an oilrig.

In many cases there is no exclusive right. Let us return to the pulp investment considered. This is, in everyday language, a right (i.e. an option) to invest. However, it is <u>not</u> an option in an economic sense. If you choose not to invest, your right to invest cannot be sold to a competitor. The competitors can invest themselves if they so wish and are not prepared to pay you for not investing. Therefore, the traditional net present value rule also leads to the optimal decision.[33]

It should be pointed out that allowing for imperfect markets makes the decision rule less clear cut. Even if there is no option value of waiting, in the sense that the right to invest can be sold, this does not necessarily mean that one should invest as soon as the net present value exceeds zero. Normally, if investing now means that the opportunity to invest in the future is gone, this opportunity has a market value, for example a patent or a concession. However, if markets are imperfect, waiting may have a value for you but nobody else. If available investment capital is scarce, for example, or if investing depletes an asset, it may serve to wait for the optimality condition to be satisfied, before investing.

To summarise the exposé in this section: Different operating options have little significance in the pulp industry. No surprise, really, as pulp is a standard commodity and the manufacturing of it a mature business. Changing the degree of utilisation captures the main optionlike characteristics of pulp production, even though this is secondary to the specification of the price process.

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[33] Following Dixit and Pindyck (1994), the optimal decision rule is to invest when the value of the investment opportunity equals the value of the investment minus the investment cost, $F(P^*) = V(P^*) - \mathrm{I}$. With no value of the investment opportunity, $F(P) = 0$, we get $V(P^*) = \mathrm{I}$. It is optimal to invest when the price is such that the investment cost is covered.

8. Conclusions

Companies do not always behave as price takers. Sometimes they actively attempt to increase the price. One measure is to reduce supply through a reduction in plant utilisation. This behaviour strains the net present value criteria, since the price-risk no longer represents the riskiness of cash flow. Option pricing methods can be used in the case where the output is a traded asset, but is it worth the trouble?

Judging from this study of the pulp industry, it is presumably not worthwhile to model a variable utilisation of capacity and it should not be given the highest of priorities. The difference between a fixed and variable production rate is crucially dependent upon the price process specified. The higher the probability of really low prices the bigger the difference. The geometric Brownian motion, as seen in diagram 6.1, gives an extremely wide range and thereby a greater difference in value.

What is important, though, is to specify the price process carefully and merely performing a net present value calculation is not enough. In fact, as it is compatible with the geometric Brownian motion assumption, its ability to adequately represent the price process over a 30-year time span is appalling. Using the mean reverting process here suggested and applying the Feynman-Kac formula in order to evaluate capital investments therefore seems attractive. Modelling different production policies then becomes less interesting as the difference in value will be small. The problem of parameter estimation and the mathematics involved, reduces the usefulness of the mean reverting process and the Feynman-Kac formula for everyday capital budgeting. It is however very suitable for major long-run investments.

Appendix 1 - Congruence between present value and real option calculations

When the price P develops according to the process $dP = \alpha P dt + \sigma(t, P)dw$, where α is a constant, the Feynman-Kac formula will give the same answer as a present value calculation for all payments that are symmetric in P. The key to this congruence, is the fact that the expected value of the above stochastic process is $E[P(t)] = P_0 e^{\alpha t}$. The stochastic term disappears, because the expected value of a deterministic Ito integral is zero.

Denote the future payment $f(P)$. That a payment is symmetric in P, is the same as saying that the payment is linear in P, i.e. $f(P) = aP$. The present value of $f(P)$ is,

$$PV = e^{-\mu t} E[f(P)] = e^{-\mu t} E[aP(t)] = e^{-\mu t} aP_0 e^{\alpha t}.$$

As is discussed in Section 2, paper pulp not is an investment object but a commodity. The total return from holding the pulp can be subdivided into the expected increase in price α and the convenience yield δ. Mathematically, $\mu = \alpha + \delta$, giving $-\mu + \alpha = -\delta$, and the present value as

$$PV = aP_0 e^{-\delta t}.$$

This is the same result as would have been obtained with the Feynman-Kac formula. With a drift rate of $r - \delta$, today's value of the future payment becomes

$$V_0 = e^{-rt} E^Q[f(P)] = e^{-rt} E^Q[aP(t)] = e^{-rt} aP_0 e^{(r-\delta)t} = aP_0 e^{-\delta t},$$

which is the same as the present value calculation.

Appendix 2 - Market data, geometric Brownian motion

Market data is collected quarterly[34] for the 1980-96 period and is reproduced at the next page. Pulp prices have been obtained from OM Stockholm AB and refers to the market price for Northern Bleached Softwood Kraft pulp. The market price is established in U.S. dollars, and has been converted to Swedish crowns by the exchange rate given in "*Main Economic Indicator*", published by the OECD Statistics Directorate.

Inflation is measured by the Swedish Consumer Price Index, and the return on 3-month Treasury Bills serves as proxy for the instantaneous risk-free rate. The market portfolio is represented by "Affärsvärldens generalindex" at the Stockholm Stock Exchange. These data are obtained through the Hanson&Partner database Ecowin, a macroeconomic database.

All parameters are continuously compounded, and the drift and standard deviation of the geometric Brownian motion is calculated by means of the procedure described in Section 2 of the main text. The results are:

Real interest rate = $\frac{1}{n}\sum_t \left[r_t - \ln\left(\frac{CPI_t}{CPI_{t-4}}\right)\right] = 4.4\%$

Inflation = $4\left[\frac{1}{n}\ln\left(\frac{CPI_n}{CPI_1}\right)\right] = 5.8\%$

Beta of pulp prices = $\frac{Cov(pulp, market)}{Var(market)} = \frac{\sum_t \left[\ln\frac{Pulp_t}{Pulp_{t-1}} - \frac{1}{n}\ln\frac{Pulp_n}{Pulp_1}\right]\left[\ln\frac{Index_t}{Index_{t-1}} - \frac{1}{n}\ln\frac{Index_n}{Index_1}\right]}{\sum_t \left[\ln\frac{Index_t}{Index_{t-1}} - \frac{1}{n}\ln\frac{Index_n}{Index_1}\right]^2} = 0.164$

Standard deviation of pulp prices, $\sigma = \sqrt{4\frac{1}{n}\sum_t \left[\ln\frac{Pulp_t}{Pulp_{t-1}} - \frac{1}{n}\ln\frac{Pulp_n}{Pulp_1}\right]^2}$ = 18.9% (real data)

= 18.9% (nominal data)

Drift of pulp prices, $\alpha = 4\left[\frac{1}{n}\ln\left(\frac{CPI_n}{CPI_1}\right)\right] + \frac{1}{2}\sigma^2$ = -0.7% (real data)

= 5.0% (nominal data)

[34] Thereby the factor 4 in the formulas on this page.

Year	NBSK USD/ton	F/X SEK/USD	CPI kr 1980=100	Interest rate 3 month	Stockholm share prices	NBSK SEK/ton	NBSK REAL 1996 SEK/ton	Interest rate cont. compounded
80	500	4,457	97,1	0,105	104,4	11219	29450	0,1036
	545	4,150	98,4	0,123	107,1	13132	34017	0,1211
	545	4,163	102,6	0,125	103,8	13091	32523	0,1231
	545	4,373	105,2	0,124	122,6	12464	30199	0,1216
81	545	4,592	109,8	0,151	142,4	11867	27550	0,1482
	545	5,085	111,6	0,135	167,2	10718	24480	0,1328
	545	5,598	114,3	0,105	166,2	9736	21712	0,1036
	545	5,571	114,9	0,090	192,3	9783	21703	0,0890
82	545	5,951	119,3	0,128	185,1	9158	19568	0,1257
	520	6,092	121,1	0,136	183,5	8536	17967	0,1339
	460	6,290	122,9	0,145	202,9	7313	15167	0,1424
	420	7,294	125,9	0,123	259,6	5758	11657	0,1213
83	400	7,509	129,3	0,109	355,7	5327	10501	0,1077
	440	7,642	131,8	0,112	367,1	5757	11135	0,1106
	440	7,822	134,5	0,117	425,0	5625	10661	0,1148
	440	8,001	137,5	0,117	430,3	5499	10194	0,1157
84	465	7,716	140,9	0,108	476,1	6026	10902	0,1061
	540	8,184	142,4	0,118	431,3	6598	11811	0,1163
	540	8,584	144,8	0,134	406,1	6291	11074	0,1321
	460	8,990	148,8	0,117	382,3	5117	8766	0,1150
85	415	8,893	152,1	0,138	387,7	4667	7821	0,1352
	390	8,804	153,9	0,163	367,4	4430	7337	0,1593
	390	8,065	154,5	0,148	381,8	4836	7979	0,1456
	400	7,616	157,1	0,124	479,7	5252	8522	0,1218
86	415	7,322	158,7	0,108	574,5	5668	9103	0,1061
	450	7,116	159,7	0,098	663,9	6323	10093	0,0971
	480	6,903	161,3	0,087	703,1	6954	10989	0,0865
	520	6,819	162,3	0,091	724,5	7626	11977	0,0903
87	550	6,327	164,7	0,108	756,0	8693	13453	0,1063
	585	6,388	164,9	0,088	803,0	9158	14156	0,0870
	610	6,438	169,4	0,090	951,7	9475	14257	0,0888
	635	5,848	170,7	0,091	667,5	10859	16215	0,0897
88	680	5,878	173,7	0,094	790,5	11569	16977	0,0932
	725	6,254	176,3	0,102	851,8	11593	16762	0,1011
	760	6,434	178,8	0,104	907,2	11812	16839	0,1027
	760	6,157	180,9	0,104	1013,8	12344	17393	0,1030
89	810	6,425	184,7	0,114	1129,1	12608	17400	0,1128
	840	6,648	187,9	0,116	1225,0	12636	17142	0,1140
	840	6,409	190,2	0,116	1285,1	13107	17565	0,1144
	840	6,227	192,8	0,123	1262,0	13490	17834	0,1210
90	840	6,126	205,4	0,146	1142,2	13713	17018	0,1438
	840	6,041	206,2	0,126	1309,7	13905	17188	0,1237
	800	5,764	212,0	0,131	910,0	13879	16688	0,1286
	750	5,698	213,9	0,144	870,0	13163	15685	0,1411
91	700	6,091	225,8	0,121	1093,7	11493	12975	0,1192
	600	6,546	227,0	0,106	1130,9	9166	10293	0,1047
	520	6,067	229,2	0,103	1035,3	8571	9532	0,1014
	500	5,529	230,8	0,136	917,6	9043	9987	0,1340
92	540	5,977	231,3	0,117	999,9	9035	9957	0,1153
	560	5,513	231,5	0,116	913,0	10158	11185	0,1143
	580	5,292	234,6	0,201	696,7	10960	11908	0,1964
	500	7,043	234,9	0,106	912,6	7100	7704	0,1044
93	470	7,745	242,7	0,097	994,5	6068	6373	0,0959
	440	7,707	242,3	0,083	1083,0	5709	6006	0,0823
	410	8,041	244,5	0,077	1294,8	5099	5316	0,0759
	400	8,304	244,3	0,071	1402,8	4817	5026	0,0700
94	440	7,828	246,8	0,071	1403,6	5621	5805	0,0699
	535	7,690	248,4	0,071	1372,4	6957	7139	0,0700
	605	7,488	250,7	0,079	1412,4	8079	8215	0,0783
	700	7,462	250,4	0,081	1470,8	9381	9550	0,0806
95	750	7,372	253,3	0,087	1458,6	10173	10237	0,0857
	858	7,268	255,3	0,093	1643,0	11804	11786	0,0923
	925	6,905	256,2	0,088	1842,3	13396	13328	0,0871
	940	6,658	256,0	0,084	1735,7	14118	14057	0,0832
96	650	6,696	257,0	0,068	1898,0	9708	9628	0,0675
	550	6,652	256,3	0,057	1981,6	8269	8224	0,0562
	580	6,628	256,0	0,047	2091,3	8751	8713	0,0466
	560	6,871	254,9	0,036	2402,9	8150	8150	0,0357

Table A.2.1, Market data for pulp parameter estimations.

Appendix 3 - Derivation of the mean reverting process

Assume the commodity price to follow the mean reverting process

$$dP = \eta(\gamma + \omega t - \ln P)Pdt + \sigma P dw \,. \tag{A.3.1}$$

Define $X(t) = \ln P(t)$ and apply Ito's lemma

$$\begin{aligned} dX &= \left[X_t + \eta(\gamma + \omega t - \ln P)PX_P + \tfrac{1}{2}\sigma^2 P^2 X_{PP}\right]dt + \sigma P X_P \, dw \\ &= \left[0 + \eta(\gamma + \omega t - \ln P)P\frac{1}{P} - \tfrac{1}{2}\sigma^2 P^2 \frac{1}{P^2}\right]dt + \sigma P \frac{1}{P} dw \\ &= \left[\eta(\gamma + \omega t - X) - \tfrac{1}{2}\sigma^2\right]dt + \sigma \, dw \,. \end{aligned}$$

The stochastic variable X follows the process

$$dX = \eta(\gamma' + \omega t - X)dt + \sigma dw \quad \text{with} \quad \gamma' = \gamma - \frac{\sigma^2}{2\eta} \,, \tag{A.3.2}$$

which is an ordinary Ornstein-Uhlenbeck process if the equilibrium drift rate ω is zero. In order to write the process of $X(T)$ in explicit form, consider the function

$$f(t, X) = e^{-\eta(T-t)} X(t)$$

with the partial derivatives

$$f_t = \eta e^{-\eta(T-t)} X \; f_X = e^{-\eta(T-t)} \; f_{XX} = 0 \,.$$

Applying Ito's lemma gives

$$\begin{aligned} df &= \left[f_t + \eta(\gamma' + \omega t - X) f_X + \tfrac{1}{2}\sigma^2 f_{XX}\right]dt + \sigma \, f_X \, dw \\ &= \left[\eta e^{-\eta(T-t)} X + \eta(\gamma' + \omega t - X) e^{-\eta(T-t)}\right]dt + \sigma \, e^{-\eta(T-t)} dw \\ &= \eta(\gamma' + \omega t) e^{-\eta(T-t)} dt + \sigma \, e^{-\eta(T-t)} dw \end{aligned}$$

Formally, this diffusion process represents the integral equation

$$f(T) = f_{t_0} + \int_{t_0}^{T} \eta(\gamma' + \omega t) e^{-\eta(T-t)} dt + \int_{t_0}^{T} \sigma e^{-\eta(T-t)} dw(t)$$

$$X(T) e^{-\eta(T-T)} = X(t_0) e^{-\eta(T-t_0)} + \int_{t_0}^{T} \eta(\gamma' + \omega t) e^{-\eta(T-t)} dt + \int_{t_0}^{T} \sigma e^{-\eta(T-t)} dw(t)$$

$$X(T) = X(t_0) e^{-\eta(T-t_0)} + \int_{t_0}^{T} \eta(\gamma' + \omega t) e^{-\eta(T-t)} dt + \int_{t_0}^{T} \sigma e^{-\eta(T-t)} dw(t) \,. \tag{A.3.3}$$

In order to derive the distribution of $X(T)$, a useful lemma from stochastic calculus will be used:

If $h(t)$ is a deterministic function of time, and the process $Y(T)$ is defined as

$$Y(T) = \int_{t_0}^{T} h(t)dw(t),$$

then $Y(T)$ is normally distributed with zero mean and the variance

$$Var[Y(T)] = \int_{t_0}^{T} h^2(t)dt.$$

This result can, for example, be found in Björk (1998) pp 43.

Applied to the process X, we immediately get

$$E[X(T)] = X(t_0)e^{-\eta(T-t_0)} + \int_{t_0}^{T} (\gamma' + \omega t)\eta\, e^{-\eta(T-t)}dt.$$

Solving the integral through integration by parts,

$$\begin{aligned} E[X(T)] &= X(t_0)e^{-\eta(T-t_0)} + \left[(\gamma' + \omega t)e^{-\eta(T-t)}\right]_{t_0}^{T} - \int_{t_0}^{T} \omega\, e^{-\eta(T-t)}dt \\ &= X(t_0)e^{-\eta(T-t_0)} + \gamma' + \omega T - (\gamma' + \omega t_0)e^{-\eta(T-t_0)} - \left[\frac{\omega}{\eta}e^{-\eta(T-t)}\right]_{t_0}^{T} \\ &= X(t_0)e^{-\eta(T-t_0)} + \omega\left(T - t_0 e^{-\eta(T-t_0)}\right) + \left(\gamma' - \frac{\omega}{\eta}\right)\left(1 - e^{-\eta(T-t_0)}\right). \end{aligned} \tag{A.3.4}$$

The expression for the variance is directly obtained from the lemma above.

$$Var[X(T)] = \int_{t_0}^{T} \sigma^2 e^{-2\eta(T-t)}dt = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta(T-t_0)}\right) \tag{A.3.5}$$

Altogether, the integral equation of (A.3.3) can be written as

$$X(T) = X(t_0)e^{-\eta(T-t_0)} + \omega\left(T - t_0\, e^{-\eta(T-t_0)}\right) + \left(\gamma' - \frac{\omega}{\eta}\right)\left(1 - e^{-\eta(T-t_0)}\right) + \sigma\sqrt{\frac{1 - e^{-2\eta(T-t_0)}}{2\eta}}\,\varepsilon, \tag{A.3.6}$$

where ε is a random drawing from a standardised normal distribution.

Summing up:

The problem of explicitly solving the mean reverting process

$$dP = \eta(\gamma + \omega t - \ln P)P dt + \sigma P dw$$

can by defining $X(t) = \ln P(t)$ be reduced to solving the process

$$dX = \eta(\gamma' + \omega t - X) dt + \sigma dw \quad \text{where} \quad \gamma' = \gamma - \frac{\sigma^2}{2\eta}.$$

$X(T)$ is then normally distributed with

$$E[X(T)] = X(t_0)e^{-\eta(T-t_0)} + \gamma' + \omega T - (\gamma' + \omega t_0)e^{-\eta(T-t_0)} - \frac{\omega}{\eta}\left(1 - e^{-\eta(T-t_0)}\right)$$

and

$$Var[X(T)] = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta(T-t_0)}\right).$$

Appendix 4 - Parameter estimation of the mean reverting process

There is no standard procedure for estimating the parameters of a continuous time mean reverting processes. One procedure, hinted at by Dixit and Pindyck (1994) pp. 76, is to use the AR1 process,

$$\frac{P(t_{k+1})-P(t_k)}{P(t_k)}=(\gamma+\omega t_k)\left(1-e^{-\eta\Delta t}\right)+\left(e^{-\eta\Delta t}-1\right)\ln P(t_k)+\sigma\,\varepsilon_{\Delta t}\,, \qquad \text{(A.4.1)}$$

as the discrete time process on which to base parameter estimation. By using a Maclaurin expansion on the term $e^{-\eta\Delta t}$ and dropping all terms of $O(\Delta^2 t)$ and higher, it is easy to see that the AR1 process converges to the mean reverting process of this paper,

$$dP=\eta(\gamma+\omega t-\ln P)Pdt+\sigma P dw\,. \qquad \text{(A.4.2)}$$

Unfortunately, running the AR1 process (A.4.1) on the available pulp data will result in a regression without any explanatory power. Presumably, dropping all terms of $O(\Delta^2 t)$ and higher is too crude a method for the quarterly data available.

Another approach is to follow Schwartz (1997) and apply the Kalman filter methodology. This approach suffers from two drawbacks. First of all, the recursive estimation procedure necessary, is quite a complex task for non-statisticians. Secondly, the state-variable (i.e. the pulp price) is usually unobservable when the Kalman filter is applied. As pulp prices are observable, the method seems less than ideal and a slight overkill.

Better then, is to follow Harvey (1989) and use the explicit solution to the continuous time process as the base for discrete time representation. Equation (A.3.7) gives the logarithm of the pulp price as,

$$X(T)=X(t_0)e^{-\eta(T-t_0)}+\omega\left(T-t_0\,e^{-\eta(T-t_0)}\right)+\left(\gamma'-\frac{\omega}{\eta}\right)\left(1-e^{-\eta(T-t_0)}\right)+\sigma\sqrt{\frac{1-e^{-2\eta(T-t_0)}}{2\eta}}\,\varepsilon\,,$$

where ε is a random drawing from a standardised normal distribution. Defining $\Delta t=t_{k+1}-t_k$, makes it possible to discretise the process as,

$$\begin{aligned}X(t_{k+1})&=X(t_k)e^{-\eta\Delta t}+\omega\left(t_{k+1}-t_k\,e^{-\eta\Delta t}\right)+\left(\gamma'-\frac{\omega}{\eta}\right)\left(1-e^{-\eta\Delta t}\right)+\sigma\sqrt{\frac{1-e^{-2\eta\Delta t}}{2\eta}}\,\varepsilon\\&=\left(\gamma'-\frac{\omega}{\eta}\right)\left(1-e^{-\eta\Delta t}\right)+\omega\left(\Delta t+t_k-t_k\,e^{-\eta\Delta t}\right)+e^{-\eta\Delta t}X(t_k)+\sigma\sqrt{\frac{1-e^{-2\eta\Delta t}}{2\eta}}\,\varepsilon\\&=\left(\gamma'-\frac{\omega}{\eta}\right)\left(1-e^{-\eta\Delta t}\right)+\omega\Delta t+\omega\left(1-e^{-\eta\Delta t}\right)t_k+e^{-\eta\Delta t}X(t_k)+\sigma\sqrt{\frac{1-e^{-2\eta\Delta t}}{2\eta}}\,\varepsilon\,.\end{aligned} \qquad \text{(A.4.3)}$$

Parameters can then be estimated by running the regression,

$$X(t_{k+1})=c_0+c_1\,t_k+c_2\,X(t_k)+s\,\varepsilon\,.$$

Unfortunately, the problem of multicollinarity makes this regression less trustworthy. One possible solution is to set the equilibrium drift rate $\omega = 0$, and run the reduced regression

$$X(t_{k+1}) = c_0 + c_2\, X(t_k) + s\,\varepsilon\,, \qquad \text{(A.4.4)}$$

with
$$c_0 = \gamma'\left(1 - e^{-\eta\Delta t}\right)$$
$$c_2 = e^{-\eta\,\Delta t}$$
$$s = \sigma\sqrt{\frac{1 - e^{-2\eta\,\Delta t}}{2\eta}}\,.$$

The regression printout on the next page gives the intercept $c_0 = 0.50591$, the coefficient $c_2 = 0.94039$, and the random error $s = \sqrt{0.00886}$.

The parameter estimation of $\{\eta, \gamma, \gamma', \sigma\}$ will then be

$$\eta = -\frac{1}{\Delta t}\ln c_2 = -4\ln 0.94039 = 0.24584$$
$$\gamma' = \frac{c_0}{1 - c_2} = \frac{0.50591}{1 - 0.94039} = 8.48700$$
$$\sigma = \sqrt{\frac{2\eta\, s^2}{1 - e^{-2\eta\,\Delta t}}} = \sqrt{\frac{2\cdot 0.24584\cdot 0.00886}{1 - e^{-2\cdot 0.24584/4}}} = 0.19407$$
$$\gamma = \gamma' + \frac{\sigma^2}{2} = 8.48700 + \frac{0.19407^2}{2} = 8.56359$$

Year	NBSK REALT 1996 kr/ton	ln P(t)	ln P(t+1)
80	5850	8,67420	8,67578
	5859	8,67578	8,63710
	5637	8,63710	8,66118
	5774	8,66118	8,66740
81	5810	8,66740	8,75301
	6330	8,75301	8,82521
	6804	8,82521	8,81517
	6736	8,81517	8,84357
82	6930	8,84357	8,80506
	6668	8,80506	8,69977
	6002	8,69977	8,73276
	6203	8,73276	8,68633
83	5921	8,68633	8,78006
	6503	8,78006	8,78298
	6522	8,78298	8,78362
	6526	8,78362	8,77816
84	6491	8,77816	8,97598
	7911	8,97598	9,00695
	8160	9,00695	8,86556
	7084	8,86556	8,72986
85	6185	8,72986	8,64587
	5687	8,64587	8,55430
	5189	8,55430	8,50565
	4943	8,50565	8,49305
86	4881	8,49305	8,53923
	5111	8,53923	8,56331
	5236	8,56331	8,62496
	5569	8,62496	8,59151
87	5386	8,59151	8,66159
	5777	8,66159	8,68434
	5910	8,68434	8,62068
	5545	8,62068	8,67683
88	5865	8,67683	8,78800
	6555	8,78800	8,84955
	6971	8,84955	8,79382
	6593	8,79382	8,87932
89	7182	8,87932	8,93262
	7575	8,93262	8,88390
	7215	8,88390	8,84152
	6916	8,84152	8,76179
90	6386	8,76179	8,74403
	6273	8,74403	8,62053
	5544	8,62053	8,53555
	5093	8,53555	8,47903
91	4813	8,47903	8,39167
	4410	8,39167	8,16295
	3509	8,16295	8,02402
	3053	8,02402	8,17663
92	3557	8,17663	8,13128
	3399	8,13128	8,11224
	3335	8,11224	8,24832
	3821	8,24832	8,24887
93	3823	8,24887	8,17954
	3567	8,17954	8,14239
	3437	8,14239	8,15063
	3466	8,15063	8,17675
94	3557	8,17675	8,34801
	4222	8,34801	8,43520
	4606	8,43520	8,57866
	5317	8,57866	8,62412
95	5564	8,62412	8,73659
	6227	8,73659	8,75697
	6355	8,75697	8,73743
	6232	8,73743	8,37023
96	4317	8,37023	8,19930
	3638	8,19930	8,24999
	3828	8,24999	8,25524
	3848	8,25524	

Regression statistics	
Multipel-R	0,92867
R-squared	0,86243
Adjusted R-squared	0,86032
Standarderror	0,09412
Observations	67

ANOVA

	DF	*SS*	*MS*	*F-value*
Regression	1	3,60990	3,60990	407,50
Residual	65	0,57581	0,00886	
Total	66	4,18572		

	Estimate	*Standarderror*	*t-ratio*
Intercept	0,50591	0,40040	1,26
X-variabel 1	0,94039	0,04658	20,19

Table A.4.1, Regression printout for the mean reverting process.

Appendix 5 - Variable production rate under mean reversion

The risk-neutral process associated with the mean reverting process specified in Section 6, is characterised by

$$\ln P(t) = X(t) \sim N\left[a(t), b(t)\right]$$

where $$a(t) = (\ln 4500)e^{-0.25t} + 0.013t + \left(8.44 - \frac{0.013}{0.25}\right)\left(1 - e^{-0.25t}\right)$$

$$b(t) = 0.19\left(\frac{1 - e^{-2\cdot 0.25t}}{2 \cdot 0.25}\right)^{1/2}.$$

The probability density function is denoted by $\varphi(x) = \frac{1}{b(t)\sqrt{2\pi}} e^{-\frac{(x-a(t))^2}{2b^2(t)}}$.

Price > 3500

An expected increase in price (and costs) of 1.3% per annum, requires the lower bound to be specified as $P(t) \geq 3500\, e^{0.013t}$. Using *X* as the stochastic variable (since it is normally distributed) with $P(t) = e^X$, the lower boundary for *X* becomes:

$$X \geq \ln 3500 + 0.013t.$$

The upper boundary of a normally distributed variable is, of course, infinity, but for computationally convenience we confine the boundary to five standard deviations.

$$V_0 \text{ (pulp sales)} = \int_0^{30} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (0.4 \cdot e^x) \cdot \varphi(x)\, dx\, dt = 24751$$

$$V_0 \text{ (pulpwood cost)} = -\int_0^{30} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (0.3 \cdot 0.4 \cdot e^x) \cdot \varphi(x)\, dx\, dt = -7425$$

$$V_0 \text{ (other variable costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (0.4 \cdot 1250 e^{0.013t}) \cdot \varphi(x)\, dx\, dt = -6345$$

V₀ (maintenance) =

$$-\int_0^{15} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (150 e^{0.013t}) \cdot \varphi(x)\, dx\, dt - \int_{15}^{30} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (250 e^{0.013t}) \cdot \varphi(x)\, dx\, dt = -2295$$

$$V_0 \textit{ (other fixed costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln(3500)+0.013t}^{a(t)+5b(t)} (300e^{0.013t}) \cdot \varphi(x)dxdt = -3807$$

2600 < Price < 3500

As the price decreases, so does the production rate. Utilisation decreases linearly from 100% for price of 3500 to only 70% for a price of 2600. Not only production and variable costs are reduced within this price range. Also maintenance is cut. It is possible to cut down on maintenance since maximum output is not an issue. Even if the plant is out of operation for a while, this is no major issue since it is possible to catch up on production later.

Denote the level of utilisation with *f(P).* In nominal terms, utilisation changes linearly from 70% when the price equals $2600e^{0.013t}$ to 100% when the price is $3500e^{0.013t}$. Utilisation as a function of price will then be the straight line $f(P) = \frac{e^{-0.013t}}{3000}P - 0.167$. Expressed in the variable *X* instead, the utilisation function becomes $f(X) = \frac{e^{-0.013t}}{3000} \cdot 4500e^{X} - 0.167$.

The integration limits for the stochastic variable *X*, is $\ln 2600 + 0.013t \le X \le \ln 3500 + 0.013t$.

$$V_0 \textit{ (pulp sales)} = \int_0^{30} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} f(x) \cdot \left(0.4 \cdot e^{x}\right) \cdot \varphi(x)dxdt = 2644$$

$$V_0 \textit{ (pulpwood cost)} = -\int_0^{30} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} f(x) \cdot \left(0.3 \cdot 0.4 \cdot e^{x}\right) \cdot \varphi(x)dxdt = -793$$

$$V_0 \textit{ (other variable costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} f(x) \cdot \left(0.4 \cdot 1250e^{0.013t}\right) \cdot \varphi(x)dxdt = -1043$$

V_0 *(maintenance)* =

$$-\int_0^{15} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} f(x) \cdot \left(150e^{0.013t}\right) \cdot \varphi(x)dxdt - \int_{15}^{30} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} f(x) \cdot \left(250e^{0.013t}\right) \cdot \varphi(x)dxdt$$

= - 387

$$V_0 \textit{ (other fixed costs)} = -\int_0^{30} e^{-0.064t} \int_{\ln(2600)+0.013t}^{\ln(3500)+0.013t} \left(300e^{0.013t}\right) \cdot \varphi(x)dxdt = -710$$

Price < SEK 2600

Changing the price variable gives the upper integration limit as $X \leq \ln 2600 + 0.013t$.
The lower integration limit of minus infinity is for computational convenience confined to five standard deviations, $a(t) - 5b(t)$. Only fixed costs are present when the price is less than 2600 Swedish crowns and the present value becomes

$$V_0 \textit{ (other fixed costs)} = -\int_0^{30} e^{-0.064t} \int_{a(t)-5b(t)}^{\ln(2600)+0.013t} \left(300e^{0.013t}\right) \cdot \varphi(x)\,dx\,dt = -91$$

Totally

Added together, V_0 *(future cash flow)* = 4499MSEK. Changing the integration limits so that full production is sustained for a price exceeding SEK 2600 per ton, gives V_0 *(future cash flow)* = 4540 MSEK.

Appendix 6 - The expansion option based on a price average

Valuation of path dependent contracts, such as average options, is perfectly feasible within the already established framework of risk-neutral valuation, even though this cannot be verified by the derivation provided in this paper. See instead the "equivalent martingale measure approach", for example in Björk (1994).

We are to value a contract (the expansion option in Section 7) that pays SEK 590 million in the case that the geometric average of the pulp price between years 20 and 30 is above a specified price. Using risk-neutral valuation, we are to find the probability density function of the contract and given the geometric Brownian motion, $dP = (r-\delta)Pdt + \sigma\, Pdv$, this is analytically feasible.

Letting the averaging period start at t_1 and the average be observed n times $\{t_2, t_3, t_4, ..., t_{n-1}, T\}$, the geometric average becomes

$$G_1(T) = \left[P(t_2)\cdot P(t_3)\cdot P(t_4)\cdot ...\cdot P(T)\right]^{1/n}.$$

Using the solution to the geometric Brownian motion,

$$P(t+1) = P(t)e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon}, \tag{A.6.1}$$

where $\varepsilon \sim N(0,1)$, we can through iteration express $G_1(T)$ as

$$\begin{aligned} G_1(T) = [&P(t_1)e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_1} \\ &\cdot P(t_1)e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_1}\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_2} \\ &\cdot P(t_1)e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_1}\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_2}\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_3} \\ &\vdots \\ &\cdot P(t_1)e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_1}\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_2}\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_3}\cdot ...\cdot e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}\,\varepsilon_n}\,]^{1/n} \end{aligned}$$

Noting that there are n rows and n columns above makes it possible to shorten the notation to

$$G_1(T) = P(t_1)\left[e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)\Delta t\sum_{i=1}^{n} i\;+\sigma\sqrt{\Delta t}\sum_{i=1}^{n} i\varepsilon_i}\right]^{1/n}.$$

Taking $1/n$ inside the parenthesis and writing $\Delta t = \frac{T-t_1}{n}$ gives the geometric average as

$$G_1(T) = P(t_1)\cdot e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)(T-t_1)\frac{1}{n^2}\sum_{i=1}^{n} i\;+\sigma(T-t_1)^{1/2}\frac{1}{n^{3/2}}\sum_{i=1}^{n} i\varepsilon_i}.$$

Except for notation, the derivation so far has followed Turnbull and Wakeman (1991). For analytical tractability and ease of application, we now allow for continuous observations by letting *n* go to infinity.

Using the results,

$$\lim_{n\to\infty} \frac{1}{n^2}\sum_{i=1}^{n} i \quad = \lim_{n\to\infty} \frac{1}{n^2}\frac{n(n+1)}{2} = \frac{1}{2}$$

and[35]

$$\lim_{n\to\infty} \frac{1}{n^3}\sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{1}{3},$$

the geometric average becomes

$$G_1(T) = P(t_1)\cdot e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)\left(\frac{T-t_1}{2}\right)+\sigma\left(\frac{T-t_1}{3}\right)^{1/2}\varepsilon}.$$

This result can also be found in Kemna and Vorst (1990), although they use another derivation.

To find the distribution of the contract, it is necessary to express $P(t_1)$ as a function of the price today as we want t_1 to be 20 years ahead. This can be done by writing $P(t_1)$ in the same form as equation (A.6.1), giving

$$G(T) = P_0\cdot e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)\left(\frac{T+t_1-2t_0}{2}\right)+\sigma\left(\frac{T+2t_1-3t_0}{3}\right)^{1/2}\varepsilon}.$$

As SEK 590 million is the payoff if the next generation's pulp mill is built, we can write the expansion option as

$$V_0(\textit{expansion option}) = e^{-0.064\cdot 30}\int_{\textit{cut off price}}^{a+5b} 590e^{0.013\cdot 30}\cdot\varphi(x)dx \qquad \text{(A.6.2)}$$

with

$$\varphi(x) = \frac{1}{b\sqrt{2\pi}}e^{-\frac{(x-a)^2}{2b^2}}$$

$$a = \left(r-\delta-\tfrac{1}{2}\sigma^2\right)\left(\frac{T+t_1-2t_0}{2}\right) = \left(0.064-0.064-\tfrac{1}{2}\cdot 0.189^2\right)\left(\frac{30+20}{2}\right) = -0.447 \qquad \text{(A.6.3)}$$

$$b = \sigma\left(\frac{T+2t_1-3t_0}{3}\right)^{1/2} = 0.189\left(\frac{30+2\cdot 20}{3}\right)^{1/2} = 0.913.$$

---

[35] *i* and *n* are squared in the summation below as it is the variance of the normal distribution that can be summed.

Without averaging, the cut off price was set as SEK 4000 per ton multiplied by the expected increase in spot price. Using the same logic, we set the cut off price to SEK 4000 per ton multiplied by the expected increase in the average price (between the years 20-30).

Changing the drift rate in a risk-neutral world r-δ, to the (spot) drift rate in the real world α, gives a = -0.122 in equation (A.6.3). We therefore get the cut off price as,

$$\textit{Cut off price} = E[G(T)] = 4000 \cdot \int_{-\infty}^{\infty} \frac{e^x}{b\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2b^2}} dx = 4000 \cdot e^{a+\frac{1}{2}b^2} = 5374.$$

Switching back to the risk neutral world, the value of the expansion option A.6.2 becomes

$$V_0\textit{(expansion option)} = e^{-0.064 \cdot 30} \int_{\ln\left(\frac{5374}{4500}\right)}^{a+5b} 590 e^{0.013 \cdot 30} \cdot \varphi(x) dx = 32.$$

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