The anatomy of a business game

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We describe in detail a business game, which has been used extensively in education for a decade. Although the business game is smaller than other games, it is fairly realistic as it includes decisions on investments, production, prices, and advertising. Furthermore, the game has dynamic properties, in that decisions and financial states of the firms carry over from one period to the next. There are not many so detailed descriptions of business games, although this is in demand. Such a complete mathematical description lays ground not only for alterations of the game, but also for developments of new games. It can also provide a link to models used in micro-economic theory.

1 Introduction

Business games are played to give participants a better understanding of how markets work and how to make decisions. The games are often designed to be fairly realistic models of industries in business life. Therefore, business games have a number of decision variables and also dynamic properties.

In the related fields of microeconomics and industrial organization (Tirole, 1986), game models are described in detail. Furthermore, the games are used for experiments (Holt, 1995). The purpose of the experiments is to study the decisions made. As these models are described in detail, the best decisions can be calculated and used as benchmarks when studying decisions made in the experiments. However, in economics, there have for a number of years been calls for more realistic, and therefore also more complex, models with dynamic properties (from, for example Shubik, 1975, to Vives, 1999). The relative realism of business games makes them a good complement to models in economics.

We shall here describe a business game (Ståhl, 1986) in detail. The game has been used extensively for over a decade as it has been played regularly as part of courses at different universities and in executive training in the US, Sweden, England, France, Russia and the Baltic states.

It is played for 3-4 hours, including briefing and debriefing. The game is smaller than other business games, which typically are played for days. Therefore, it enables us to describe it in full detail, using mathematical formulas, without being too lengthy. The business game has resemblance to some models listed in Gold (2001). Two of the few examples in economics of similar models are Stigler (1968), where the decision variable advertising is referred to as a non-price competition variable, and a marketing model in Shubik & Levitan (1980). These models are, however, static.

To our knowledge, there are few complete mathematical descriptions of business games. Such descriptions can improve the scientific knowledge of gaming/simulation (Wolfe & Crookall 1998) and provide the ground for new and improved games. The mathematical description of the game also allows the determination of theoretical solutions of the game and might also lay ground for how other business games can be described.

First, we shall describe the business game generally. Then, we describe the game in detail, mathematically. Finally, we discuss how this detailed description can be used.

2 General description[1]

The business game deals with an oligopoly market where firms compete, by producing and selling similar, but not identical, storable products. The objective of a firm is to maximize the equity at the end of the game.

The game has dynamic properties, as the following four state variables are carried over from one period to the next:

- Machine capacity
- Stocks
- Balance on checking account
- Cumulative advertising

The variables above represent the state of each firm in the different periods of the game. Equity is calculated as the value of machine capacity and stocks plus the balance on the checking account. The cumulative advertising is not included in the equity, as presented in the balance sheet of the game[2].

All firms start with the same amount of equity, consisting only of cash on the checking account. Hence, the firms have initially no machine capacities, no units in stock and no cumulative advertising.

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[1] The rules of the business game are presented in appendix A.

[2] There were several reasons for not including cumulative advertising in the equity shown in the reports on the balance sheets of the firms, although from a theoretical point of view it could be regarded as part of the equity. The main reason was one of pedagogical realism. The cumulative advertising is in reality seldom included in the balance sheet or even known exactly, especially not by competitors. Furthermore, when the game is ended after a certain number of periods, the theoretical value of cumulative advertising at the end of the last period can be regarded as 0.

The firms have to decide upon the following four decision variables in each period, as the firms produce to sell their products on the market:

- Investments in machinery
- Production quantity
- Advertising
- Prices

Each unit of machinery has a fixed cost. One unit of machine capacity can produce one unit of the product in each period of the game. The cost of producing one unit is also fixed.

As the decisions are made, three outcome variables are calculated:

- Interest rate
- Demand
- Sales

Each firm has a checking account. If the balance on the checking account is negative, e.g. due to outlays on investment, production and advertising, the firms can borrow money. The interest rate is determined by an interest function. The interest rate depends on the size of the balance on the checking account and the equity of the firm. The more negative the balance and the smaller the equity, the higher the interest rate.

The demand for a firm's products in a period is dependent not only on the price and the cumulative advertising of that firm, but also on the prices and the cumulative advertising of the other firms competing on the same market. The model in the game has the characteristics of an oligopoly market, where there is *interdependence* among the firms. In this connection it should be mentioned that there are no random factors involved in the game, not even as regards the demand for the products. Thus, the state variables and the decisions of the firms completely determine the outcome.

The machines depreciate each period, both physically and in accounting terms. Products not sold in one period go into stocks and can be sold in a subsequent period. Cumulative advertising consists of advertising in a period, plus a part of the cumulative advertising from the previous period, plus a factor reflecting the advertising effect of sales in the previous period. The profit is the difference between the equity at the end of a period and the equity at the start of a period.

If a firm has equity below zero, it goes into bankruptcy. The firm can then receive a money grant from the government that decreases its debt and thus increases its equity. If a firm goes into bankruptcy repeatedly, the game leader may exclude it from the game.

The game can be played with a test period, where the firm can test their decisions. The game is then restarted from “scratch” and played for 5-10 periods. Normally it is played with uncertainty about exactly how many periods will be played.

3 Detailed description

In this section, the business game is described in detail using mathematical notations. The mathematical description below is linked to the decision form used when the game is played, presented in Appendix B, and the reports, presented in Appendix C. The variables in the decision form and in the sample report are numbered with the same numbers as in the mathematical description below. Furthermore, the model below is summarized in Appendix G.

There are variables of three types: decision variables and outcome variables, both denoted with lower case letters, and state variables, denoted with upper case letters. We here use dots to represent different states of the state variables in the game.[3] For example, capacity, denoted K, can have three different states: $K, \dot{K}, \ddot{K}$. These states are:

- Variables at the start of a period are denoted with plain letters, e.g. K.
- Intermediate variables in a period are denoted with one dot, e.g. $\dot{K}$.
- Variables at the end of a period are denoted with two dots, e.g. $\ddot{K}$.

There are also a number of parameters, denoted with Greek characters.

We shall describe the model in the following five steps:

- State variables at the start of a period
- Decision variables determined in the beginning of the period
- Intermediate state variables due to these decisions
- Outcome variables determined during the period
- State variables at the end of a period

The timing of the state variables, the decision variables and the outcome variables are shown in Figure 1 below.

Start of period	**Decisions**	**Intermediate**	**Outcome**	**End of period**
K	i	$\dot{K}$		$\ddot{K}$
S	o	$\dot{S}$		$\ddot{S}$
C		$\dot{C}$	r	$\ddot{C}$
A	a	$\dot{A}$		$\ddot{A}$
	p		d, q	
E				$\ddot{E}$

- - - - | - - - - - - - - - | - - - - - - - - - | - - - - - - - - -|^^^^^^^^^^^^^^| - - - - -
"long time"

Figure 1. State variables and decision variables in the model.

[3] Please note that the use of dots in this study is different from the use of dots in other studies, where the dot is often used to indicate time derivatives.

3.1 State variables at the start of a period

The variables at the start of a period are shown in Table 1 below.

State variable at the start of a period				Parameters	Restriction
(1)	K	=	Capacity, units of machines	σ	$K \geq 0$
(2)	S	=	Stocks, in units	χ	$S \geq 0$
(3)	C	=	Checking account balance		
(4)	A	=	Cumulative advertising	μ, η	$A \geq 0$

Table 1. State variables at the start of a period

Equity is calculated as a combination of three of the four basic state variables (i.e. excluding A as mentioned in footnote 1), and the two parameters, σ and χ [4]:

(5) $$E = \sigma K + \chi S + C \geq 0$$

σ is the cost of one unit of machinery and σK the value of capacity, χ is the direct cost of producing one unit and χS the value of stocks. C is the balance on the checking account. A positive balance implies a cash surplus and a negative balance implies a loan. A firm can have negative cash as part of its equity, as long as the equity $E \geq 0$.

3.2 Decision variables in a period

In each period, the firms have to make four decisions. A general restriction is that all decision variables must be non-negative. There are some specific restrictions as shown in Table 2 below.

Decision variable				Restriction
(6)	i	=	investments in units of machinery	$i \geq 0$
(7)	o	=	production output in units	$\dot{K} \geq o \geq 0$
(8)	a	=	advertising	$a^{MAX} \geq a \geq 0$
(9)	p	=	price	$p^{MAX} > p > \chi$

Table 2. Decision variables in a period.

[4] The parameters and the restrictions on variables are summarized in Appendix F, where they are given the numerical values used in most games played up to now.

3.3 Intermediate variables in a period

At the time the decisions are made, the model has an intermediate state, where the basic state variables are transformed to intermediate state variables. The intermediate state variables have, as mentioned, the same capital letters as the state variables, but they are denoted with one dot on top of the letter.

The intermediate capacity consists of the capacity at the start of a period plus the investments in the period.

(10) $$\dot{K} = K + i$$

As shown in Table 2, the intermediate capacity poses a restriction on production, as production is restricted to capacity, i.e. $o \leq \dot{K}$.

The supply of products that can be sold on the market from the intermediate stocks is denoted $\dot{S}$. It consists of unsold units of the product from earlier periods plus the amount of units produced in the period.

(11) $$\dot{S} = S + o$$

The payments of a firm for investments, production and advertising are denoted P, where:

(12) $$P = \sigma i + \chi o + a$$

The balance on the intermediate checking account is denoted $\dot{C}$. It is an intermediate state depending on the checking account balance at the start of the period minus the payments, due to decisions made in the period.

(13) $$\dot{C} = C - P \qquad \text{i.e.} \qquad \dot{C} = C - \sigma i - \chi o - a$$

When the balance on the intermediate checking account $\dot{C}$ is positive, the firms are said to have cash. When $\dot{C}$ is negative, a loan is automatically taken up. There is a restriction on the loan, $\dot{C} \geq C^{MIN}$, where C^{MIN} is the lowest negative balance allowed on the checking account.

A, as in (4), is the cumulative advertising at the start of the present period. The advertising a in the present period is added to this cumulative advertising to form the intermediate cumulative advertising, $\dot{A}$.

(14) $$\dot{A} = A + a$$

3.4 Outcome variables in a period

There are three outcome variables: interest rate, demand and sales.

3.4.1 Interest rate

The interest rate, r, is a function of $\dot{C}$ and E, more specifically the quotient of them, $\dot{C}/E$.

(15) $\quad r = r(\dot{C}, E)$

If the checking account balance $\dot{C}$ is positive, the interest rate is given by the flat rate $r^{CASH} > 0$. If the balance is negative, a loan is taken up automatically. The interest rate is then variable in the interval $r^{MIN} \leq r \leq r^{MAX}$. The smaller the quotient $\dot{C}/E$, i.e. the higher absolute value of $\dot{C}/E$, the higher the interest rate. The interest function r is not continuous and not differentiable. When $\dot{C}$ is very close to 0, r jumps down from r^{MIN} to r^{CASH}. Figure 2 below, illustrates the interest function.

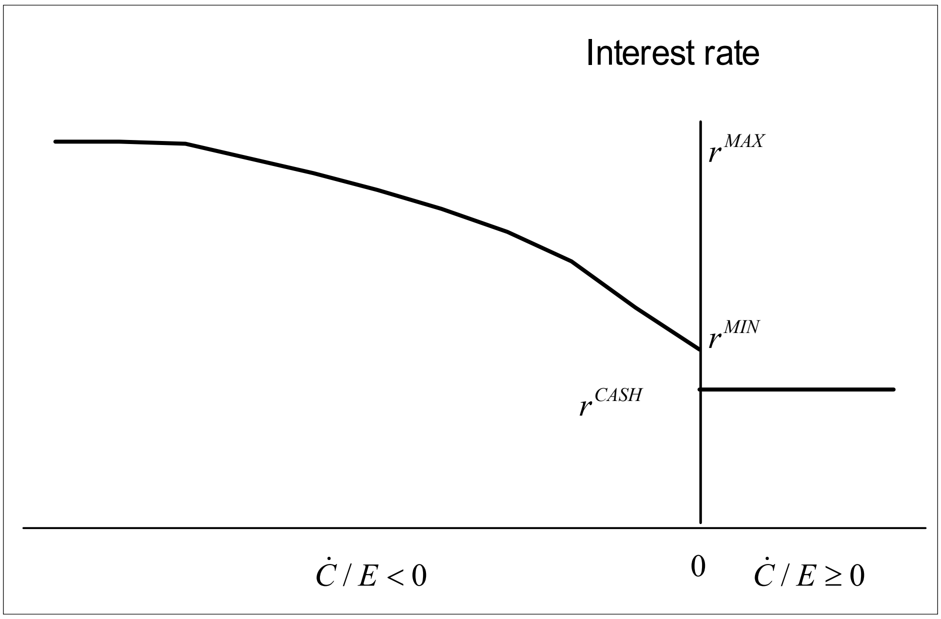

Figure 2. Interest function.

The interest function is presented as an interest table when the game is played. Appendix D contains this interest table as well as the exact interest function.

3.4.2 Demand

We next present the demand, d, for each firm's product. The demand consists of four components, denoted d_1, d_2, d_3, d_4, which are multiplied to determine the demand. We have chosen a multiplicative function both because it has appeared realistic and because it makes it easier find the optimal strategies of the game (see section 4).

The first two components, d_1 and d_2, reflect the effect of the firm's price, p, and its cumulative advertising, $\dot{A}$. d_1 implies a constant price elasticity and d_2 that the marginal effect of one additional dollar spent on advertising decreases steadily as advertising increases.[5]

(16a) $d_1 = \alpha p^{-\beta}$ (16b) $d_2 = \ln\left(\dot{A} + \varsigma\right)$

The next two components, d_3 and d_4, represent the *interdependence* among firms. We denote the total number of firms on the market N. We denote the mean price and the mean cumulative advertising of all firms, p_N and $\dot{A}_N$, respectively.

(17a) $p_N = (\sum_{j=1}^{N} p_i)/N$ and (17b) $\dot{A}_N = (\sum_{j=1}^{N} \dot{A}_i)/N$

The component d_3, interdependence of prices, depends on the difference between the price of a firm and the mean price of all N firms on the market. The component d_4, interdependence of cumulative advertising, depends on the difference between the cumulative advertising of a firm and the mean cumulative advertising of all firms on the market:

(18a) $(p_N - p)/p_N$ (18b) $(\dot{A} - \dot{A}_N)/\dot{A}_N$

The demand of a firm's product becomes higher, if its price is lower than the mean price on the market. Also, the demand for a firm's product becomes higher if its cumulative advertising is higher than the mean cumulative advertising on the market. For prices, small differences do not matter much, but large differences can have a strong effect. Therefore we assume an exponential effect for differences on prices. However, for differences in cumulative advertising we assume a proportional effect.

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[5] The parameter ς in (16b) ensures that $\ln\left(\dot{A} + \varsigma\right) \geq 0$, i.e. that $d_2 \geq 0$

(19a) $d_3 = e^{(p_N - p)/p_N}$ (19b) $d_4 = 1 + (\dot{A} - \dot{A}_N)/\dot{A}_N$

There are altogether N firms on the market and we shall use n, where is $n = N - 1$, to denote the number of competitors of the studied firm. We shall distinguish the price of this firm, p, from the mean price of the *n* other firms p_n, and the cumulative advertising of this firm, $\dot{A}$, from the mean cumulative advertising of the *n* other firms, $\dot{A}_n$.

The mean price and the mean cumulative advertising of the all firms can now be reformulated as:

(20a) $p_N = (p + np_n)/N$ (20b) $\dot{A}_N = (\dot{A} + n\dot{A}_n)/N$

For prices, we use (20a) in (18a)

(21) $(p_N - p)/p_N = 1 - p/p_N = 1 - Np/(p + np_n)$

and we get the following interdependence function for price:

(22) $d_3(p, p_n) = e^{1 - Np/(p + np_n)}$

Correspondingly for cumulative advertising, we use (20b) in (18b)

(23) $d_4 = 1 + (\dot{A} - \dot{A}_N)/\dot{A}_N = 1 + \dot{A}/\dot{A}_N - 1 = \dot{A}/\dot{A}_N$

and we get the following interdependence function for cumulative advertising:

(24) $d_4(\dot{A}, \dot{A}_n) = N\dot{A}\big/(\dot{A} + n\dot{A}_n)$

We can hence define the demand function facing a firm as dependent on its price p, its cumulative advertising $\dot{A}$ and the mean price p_n and the mean cumulative advertising $\dot{A}_n$ of the other *n* firms.

(25) $d(p, p_n, \dot{A}, \dot{A}_n) = d_1(p)d_2(\dot{A})d_3(p, p_n)d_4(\dot{A}, \dot{A}_n)$

In appendix E, there is a demand table that can be used for making a rough estimate of the demand, with the decisions of one of the firms, p and $\dot{A}$, in the two leftmost columns and the mean decisions of the other *n* firms, p_n and $\dot{A}_n$, in the top two rows.

We illustrate the demand on the market with Figure 3 below. The curves in the figure are so-called *iso-curves*, contouring all the points that refer to the demand for a given number of units. The figure is based on the parameter values in Appendix F and the assumption that all firms here have the same price and the same cumulative advertising, i.e. that we have symmetric decisions.

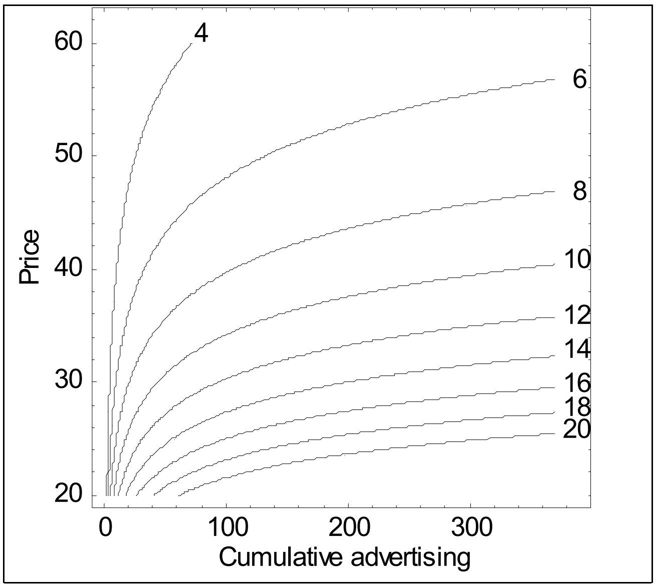

Figure 3. Demand under symmetric decisions.

3.4.3 Sales

Sales, denoted q, is the smallest of demand, d, and supply, $\dot{S}$.

(26) $q = \min[d, \dot{S}]$ (sales)

If demand is bigger than supply, i.e. $d > \dot{S}$, the difference between demand and supply is called lost sales.

(27) $d - \dot{S}$ (lost sales)

3.5 State variables at the end of a period

The four state variables are updated at the end of a period. Capacity is decreased by depreciation:

(28) $\ddot{K} = (1-\delta)\dot{K}$

Stocks are decreased by the number of units sold:

(29) $\ddot{S} = \dot{S} - q$

The checking account balance is affected by the revenue pq and the interest payment $r\dot{C}$, which is negative, if the intermediate balance on the checking account is negative, i.e. a loan is taken, and otherwise positive.

(30) $\ddot{C} = \dot{C} + pq + r\dot{C}$

The cumulative advertising is also changed. A fraction, μ, of intermediate cumulative advertising, $\dot{A}$, remains and to this is added an effect of sales, e.g. in the form of word-of-mouth influence, of having sold q units, such that each unit sold has the same effect as η spent on advertising at the end of the period.

(31) $\ddot{A} = \mu\dot{A} + \eta q$ cumulative advertising

The equity at the end of a period is, just as the equity at the start of a period (5), equal to the sum of capacity, stock and checking account balance:

(32) $\ddot{E} = \sigma\ddot{K} + \chi\ddot{S} + \ddot{C}$

Capacity, stocks, checking account and equity are at the end of the period presented in the balance sheet (see appendix C).

Since the business game is dynamic, the state variables at the end of a period are, if the game is not ended, the state variables at the start of the next period. Up to now all variables have referred to the same period and no period indices have been necessary, but here we need an index t to denote the period. Hence we have for the opening balances in the next period that:

(33a) $K_{t+1} = \ddot{K}_t$ (33b) $S_{t+1} = \ddot{S}_t$

(33c) $C_{t+1} = \ddot{C}_t$ (33d) $A_{t+1} = \ddot{A}_t$

3.6 Deriving the profit from the balance sheet

We shall derive the profit from the balance sheet (Appendix C), and specify the components of the profit. The profit of a firm consists of the equity at the end of the period minus the equity at the start of the period.

(34) $$\Pi = \ddot{E} - E$$

Using first (28), (29) and (30) in (31) and then also (10), (11), (12) and (13)

(35) $$\begin{aligned}\ddot{E} &= \sigma(1-\delta)\dot{K} + \chi(\dot{S} - q) + \dot{C} + pq + r\dot{C} = \\ &\sigma(1-\delta)(K+i) + \chi(S + o - q) + C - P + pq + r\dot{C} = \\ &\sigma K + \sigma i - \sigma\delta(K+i) + \chi S + \chi o - \chi q + C - \sigma i - \chi o - a + pq + r\dot{C} = \\ &pq - \chi q - a - \sigma\delta\dot{K} + r\dot{C} + \sigma K + \chi S + C = \\ &pq - \chi q - a - \sigma\delta\dot{K} + r\dot{C} + E\end{aligned}$$

This gives the profit in the income statement (Appendix C).

(36) $$\Pi = pq - \chi q - a - \sigma\delta\dot{K} + r\dot{C}$$

The sales revenue for each firm is calculated as:

(37) pq (revenue)

The cost components are the following three items:

(38) χq (cost of sold units)

where χ is the unit cost of production

(8) a (cost of advertising)

(39) $\delta\sigma\dot{K}$ (depreciation)

where δ is the fraction of depreciation and σ the unit price of a machine.
The last item is either revenue or cost:

(40) $r\dot{C}$ (interest payment)

where r is the interest rate and $\dot{C}$ is the intermediate balance on the checking account, i.e. after payments.

We illustrate the profit on the market with Figure 4 below. Like in Figure 3 on demand, we assume that all firms here have the same price and the same cumulative advertising, i.e. we have symmetric decisions. The curves are contouring all the points that refer to the profit of a given amount. The figure is based on the parameters in Appendix F and refers to the profits obtained in the first period.

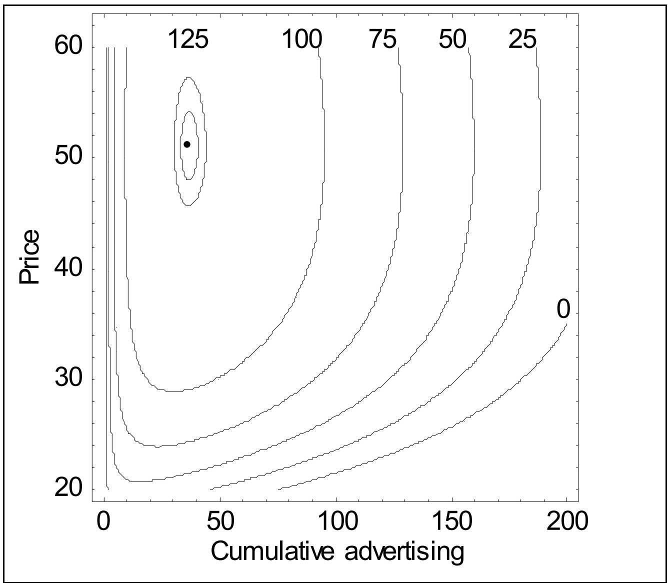

Figure 4. Profits under symmetric decisions.

4 Playing the game

The detailed description above shows, among other things, the following important concepts:

(I) The timing of state variables, decision variables and outcomes.
(II) The dynamic properties of the game, the state variables capacity, stock, checking account and cumulative advertising.
(III) The relationship between the decisions (in the decision form) and income statement and balance sheet (in the reports).
(IV) The interdependence between the firms' decisions determining the demand for their products and the demand table.
(V) The determinants of the interest rate: balance on the checking account and equity
(VI) The notion of lost sales, as the difference between demand and supply.
(VII) The determinants of profit.

A natural question is here "Are there some decisions that are more profitable than others?" (Neal 1999).

This question can be extended to the apparent and common question among the participants when playing the game: "How much profits can a firm earn in the game given it makes the best possible decisions?". This question is of importance for the learning when playing the game (Faria, 2001), as we can compare the decisions made during the play of the game to these best decisions. As we shall discuss below, the question of which decisions are the best ones is not easily answered. Furthermore, in order to be able to answer this question, we need a detailed description of the game.

As can be seen in Figure 4 above that, with symmetric decisions, a relatively high price of about 52 and a relatively low advertising of about 36 give the highest profits in the first period. Such decisions would be the decisions of a cartel that maximizes the total profit for all firms on the market in the first period. These "best decisions" are called cooperative solution or joint-maximum solution (Shubik & Levitan, 1980)

However, these decisions might not the best decisions of a single firm that maximizes its own profits. For example, one single firm could increase its profit by having lower prices and higher advertising, if the other firms stuck to the cooperative solution. If all firms make decisions only to maximize their own profits, regarding the decisions of the others as given, their best decisions would, if they also knew the decisions of the other firms, constitute what is called the non-cooperative solution or the Nash-solution (Nash, 1951 and Tirole, 1986). In this equilibrium solution, no single firm can increase its profit by changing its decision, if the other firms stick to their decisions.

Typically, the profits are lower in the non-cooperative solution than in the cooperative solution. The price would be lower and advertising higher. The Nash solution for the parameters in Appendix F would for a one-period game imply a price of about 39 and advertising of about 120. It should be mentioned that in multi-period games, which are the rule, both the cooperative and non-cooperative solutions would be somewhat different from the solutions mentioned above. The differences between the cooperative and non-cooperative solutions are, however, much bigger than the differences for each of these solutions for different number of periods[6].

It has been our experience from a great many games that the learning effect from the debriefing session at the end of the game has improved considerably when one has made comparisons between the decisions made during the game and the solutions according to these two solution concepts.

Such comparisons are also of interest for research, where we can ask the question "Which of the solutions give the best description of the decisions when the game is played". Furthermore, with a detailed description of the game, the parameters in the game can be modified to fit data from a real market. Comparisons can then be made between the decisions on the market and best decisions according to the two solutions.

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[6] The cooperative and the non-cooperative solutions for this game have been calculated numerically for different number of periods in Edman (2000).

Appendix A – The rules of the game

You compete on a market with ***n*** other firms, producing a similar, but not identical, storable product. In each period, you make four decisions:

1. Number of machines, **i**, to be purchased at **$** σ per unit
2. Number of units, **o**, to be produced at $ χ per unit
3. Amount of money, **a**, (in $) to be spent on advertising
4. Price, **p**, of the product (in $)

You can use decimals numbers for your decisions. One machine can produce one product unit. Total machine capacity sets the limit on total production. Thus, in period 1 you can only produce as much as the capacity you buy.

Machinery is subject to real depreciation; a fraction δ of the capacity breaks down each period.

At the start, each firm has 0 machines, 0 units in stock, and $ C_1 in cash on a checking account.

If there is not enough cash to cover investments, cost of production and advertising, money will automatically be borrowed. Interest rates rise with the borrowed amount as well as increased debts in relation to equity. The rate is given in the attached table.

Sales in a period depend on price and advertising policies, mainly those of the period, but to some extent also those of earlier periods. Market research indicates: If all firms in the first period charge $ ***p*** on price and spend $ ***a*** on advertising, each firms sells roughly ***d*** units.

A suitable goal might be to maximize equity at the end of the game, where Equity = Capacity * $ σ + Stock * $ χ + Checking account.

In the case that a firm obtains a negative equity, it will go bankrupt. In that case the game leader can either decide upon the firm ceasing operations, i.e. leaving the game, or being taken over by the government.

At the end of every period, each firm obtains a report on its own income statement and its own balance sheet, as well different types of reports on the other firms.

The game starts with a test period when all reports are given. The game is then restarted from period 1 again, so that the test period will not have any effect on the "real game" played over (at least) ***T*** periods.

Appendix B – Decision form

Firm number j Period t

Equity	Machine Capacity	Production and stocks	Payments	Cash (or loans)	
Production capacity	K units				(1)
Stocks		S units			(2)
Cash (if negative = loan)				\$ C	(3)
Decision 1 New investment in capacity \$ σ per unit	i units		\$ σi		(6)
Total available capacity (1)+(6)	$\dot{K} = K + i$ units				(10)
Decision 2 Production at \$ χ per unit (may not exceed (10)		o	χo		(7)
Quantity available for sales (2)+(7)		$\dot{S} = S + o$			(11)
Decision 3 Advertising			\$ a		(8)
Total payments (6)+(7)+(8)			\$ $P = \sigma i + \chi o + a$		(12)
Remaining cash (row 1 − row 8) if negative = loan			\$ $\dot{C} = C - P$		(13)
Decision 4 Price \$ p					(9)

Appendix C – Reports

Firm number j Period t

Outcome

Interest rate	r	(r depends on $\dot{C}/E$)	(15)
Sales (units)	q	(lost sales, $d-\dot{S}$ units)	(26)

Income statement

Revenues	\$	pq	(37)
Cost of goods sold	\$	χq	(38)
Advertising costs	\$	a	(8)
Interests	\$	$r\dot{C}$	(40)
Depreciation	\$	$\delta\sigma\dot{K}$	(39)
Profit	\$	$\Pi = pq - \chi q - a - \sigma\delta\dot{K} + r\dot{C}$	(36)

Balance sheet

Machine capacity	\$	$\sigma\ddot{K}$	(28)
Stocks ¤	\$	$\chi\ddot{S}$	(29)
Cash	\$	$\ddot{C}$	(30)
Equity	\$	$\ddot{E}$	(31)

Reports on other firms

	Price(9)	Sales(15)	Market share	Advertising (8)
Firm 1	p	q	$q/\sum q$	a
…				
Firm N	p	q	$q/\sum q$	a
Total(s)		$\sum q$		$\sum a$

	Capacity(16) units	Stock(17) units	Cash/Loans(18) \$	Equity (19) \$
Firm 1	$\ddot{K}$	$\ddot{S}$	$\ddot{C}$	$\ddot{E}$
…				
Firm N	$\ddot{K}$	$\ddot{K}$	$\ddot{C}$	$\ddot{E}$

Appendix D – Interest table

Interest rates and interest payments

Loan	Equity 50	100	150	200	250	300
50	16% (8)	14% (7)	13% (7)	13% (7)	13% (7)	13% (7)
100	19% (19)	16% (16)	15% (15)	14% (14)	14% (14)	13% (13)
150	21% (32)	18% (27)	16% (24)	15% (23)	14% (21)	14% (21)
200	23% (46)	19% (38)	17% (34)	16% (32)	15% (30)	15% (30)
250	24% (60)	20% (50)	18% (45)	17% (43)	16% (40)	15% (38)
300	26% (78)	21% (63)	19% (57)	18% (54)	17% (51)	16% (48)
350	27% (95)	22% (77)	20% (70)	19% (67)	18% (63)	17% (60)
400	28% (112)	23% (92)	21% (84)	19% (76)	18% (72)	17% (68)
450	28% (126)	24% (108)	21% (95)	20% (90)	19% (86)	18% (81)
500	28% (140)	24% (120)	22% (110)	20% (100)	19% (95)	18% (90)
550	28% (154)	25% (138)	23% (127)	21% (116)	20% (110)	19% (105)
600	28% (168)	26% (156)	23% (138)	21% (126)	20% (120)	19% (114)
650	28% (182)	26% (169)	24% (156)	22% (143)	21% (137)	20% (130)
700	28% (196)	27% (189)	24% (168)	22% (154)	21% (147)	20% (140)
750	28% (210)	27% (203)	24% (180)	23% (173)	21% (158)	20% (150)
800	28% (224)	28% (224)	25% (200)	23% (184)	22% (176)	21% (168)
850	28% (238)	28% (238)	25% (213)	23% (196)	22% (187)	21% (179)
900	28% (252)	28% (252)	26% (234)	24% (216)	22% (198)	21% (189)
950	28% (266)	28% (266)	26% (247)	24% (228)	23% (219)	22% (209)
1000	28% (280)	28% (280)	26% (260)	24% (240)	23% (230)	22% (220)

Figures within parentheses refer to interest payments in \$. These payments are rounded to the nearest integer in the table, but **not** in the game.

If $\dot{C} < 0$, the interest rate r is calculated from $\dot{C}$ and E in the following steps:
$l = \max(-\dot{C},1)$; $e = \max(E,0.5)$; $s = l/e$;
$if\ s \le 1\ then\ r = 0.12 + 0.04s\ else\ r = 0.10 + 0.04(\ln(2) + \ln(2s)) + 0.005s$;
$r = \min(\max(r^{MIN}, r), r^{MAX})$, where $r^{MIN} = 0.13$ and $r^{MAX} = 0.28$.

If $\dot{C} \ge 0$, $r = r^{CASH} = 0.10$.

Appendix E – Demand table

	Price	25	25	25	25	25	30	30	30	30	30	35	35	35	35	35	40	40	40	40	40	45	45	45	45	45
Price	**Cum.Adv.**	**50**	**100**	**150**	**200**	**250**	**50**	**100**	**150**	**200**	**250**	**50**	**100**	**150**	**200**	**250**	**50**	**100**	**150**	**200**	**250**	**50**	**100**	**150**	**200**	**250**
25	**50**	13.6	7.6	5.2	4.0	3.2	15.6	8.7	6.0	4.6	3.7	17.3	9.6	6.7	5.1	4.1	18.8	10.4	7.2	5.5	4.5	20.1	11.2	7.7	5.9	4.8
25	**100**	26.7	16.0	11.4	8.9	7.3	30.6	18.4	13.1	10.2	8.3	34.0	20.4	14.6	11.3	9.3	36.9	22.1	15.8	12.3	10.1	39.4	23.6	16.9	13.1	10.7
25	**150**	37.3	23.7	17.4	13.7	11.4	42.8	27.3	20.0	15.8	13.0	47.5	30.3	22.2	17.5	14.5	51.6	32.8	24.1	19.0	15.7	55.1	35.1	25.7	20.3	16.8
25	**200**	46.0	30.7	23.0	18.4	15.3	52.8	35.2	26.4	21.1	17.6	58.6	39.1	29.3	23.5	19.5	63.7	42.4	31.8	25.5	21.2	68.0	45.3	34.0	27.2	22.7
25	**250**	53.3	36.9	28.2	22.8	19.2	61.2	42.3	32.4	26.2	22.0	67.9	47.0	36.0	29.1	24.4	73.7	51.0	39.0	31.6	26.5	78.7	54.5	41.7	33.7	28.3
30	**50**	59.5	42.5	33.0	27.0	22.9	68.2	48.7	37.9	31.0	26.2	75.8	54.1	42.1	34.4	29.1	82.2	58.7	45.7	37.4	31.6	87.8	62.7	48.8	39.9	33.8
30	**100**	8.9	4.9	3.4	2.6	2.1	10.3	5.7	4.0	3.0	2.5	11.6	6.5	4.5	3.4	2.8	12.8	7.1	4.9	3.8	3.0	13.8	7.6	5.3	4.0	3.3
30	**150**	17.4	10.4	7.5	5.8	4.7	20.3	12.2	8.7	6.8	5.5	22.8	13.7	9.8	7.6	6.2	25.0	15.0	10.7	8.3	6.8	27.0	16.2	11.6	9.0	7.4
30	**200**	24.3	15.5	11.4	9.0	7.4	28.4	18.1	13.2	10.5	8.6	31.9	20.3	14.9	11.8	9.7	35.0	22.3	16.3	12.9	10.7	37.8	24.0	17.6	13.9	11.5
30	**250**	30.0	20.0	15.0	12.0	10.0	35.0	23.3	17.5	14.0	11.7	39.4	26.3	19.7	15.8	13.1	43.2	28.8	21.6	17.3	14.4	46.6	31.1	23.3	18.6	15.5
35	**50**	34.8	24.1	18.4	14.9	12.5	40.5	28.1	21.5	17.4	14.6	45.6	31.6	24.1	19.5	16.4	50.0	34.6	26.5	21.4	18.0	53.9	37.3	28.6	23.1	19.4
35	**100**	38.8	27.7	21.5	17.6	14.9	45.2	32.3	25.1	20.6	17.4	50.9	36.3	28.3	23.1	19.6	55.8	39.9	31.0	25.4	21.5	60.2	43.0	33.4	27.4	23.1
35	**150**	6.1	3.4	2.3	1.8	1.5	7.2	4.0	2.8	2.1	1.7	8.2	4.6	3.2	2.4	2.0	9.1	5.1	3.5	2.7	2.2	9.9	5.5	3.8	2.9	2.4
35	**200**	12.0	7.2	5.1	4.0	3.3	14.1	8.5	6.1	4.7	3.9	16.1	9.7	6.9	5.4	4.4	17.8	10.7	7.6	5.9	4.9	19.4	11.6	8.3	6.5	5.3
35	**250**	16.7	10.7	7.8	6.2	5.1	19.8	12.6	9.2	7.3	6.0	22.5	14.3	10.5	8.3	6.9	25.0	15.9	11.6	9.2	7.6	27.1	17.3	12.7	10.0	8.3
40	**50**	20.7	13.8	10.3	8.3	6.9	24.4	16.3	12.2	9.8	8.1	27.8	18.5	13.9	11.1	9.3	30.8	20.5	15.4	12.3	10.3	33.5	22.3	16.7	13.4	11.2
40	**100**	23.9	16.6	12.7	10.3	8.6	28.3	19.6	15.0	12.1	10.2	32.2	22.3	17.0	13.8	11.6	35.6	24.7	18.9	15.3	12.8	38.7	26.8	20.5	16.6	13.9
40	**150**	26.7	19.1	14.8	12.1	10.3	31.5	22.5	17.5	14.3	12.1	35.9	25.6	19.9	16.3	13.8	39.8	28.4	22.1	18.1	15.3	43.2	30.9	24.0	19.6	16.6
40	**200**	4.4	2.4	1.7	1.3	1.0	5.2	2.9	2.0	1.5	1.2	6.0	3.3	2.3	1.8	1.4	6.7	3.7	2.6	2.0	1.6	7.4	4.1	2.8	2.2	1.8
40	**250**	8.6	5.2	3.7	2.9	2.3	10.3	6.2	4.4	3.4	2.8	11.8	7.1	5.1	3.9	3.2	13.2	7.9	5.6	4.4	3.6	14.4	8.7	6.2	4.8	3.9
45	**50**	12.0	7.6	5.6	4.4	3.7	14.4	9.1	6.7	5.3	4.4	16.5	10.5	7.7	6.1	5.0	18.4	11.7	8.6	6.8	5.6	20.2	12.8	9.4	7.4	6.1
45	**100**	14.8	9.9	7.4	5.9	4.9	17.7	11.8	8.9	7.1	5.9	20.3	13.6	10.2	8.1	6.8	22.7	15.2	11.4	9.1	7.6	24.9	16.6	12.5	10.0	8.3
45	**150**	17.2	11.9	9.1	7.4	6.2	20.5	14.2	10.9	8.8	7.4	23.6	16.3	12.5	10.1	8.5	26.3	18.2	13.9	11.3	9.5	28.8	20.0	15.3	12.4	10.4
45	**200**	19.1	13.7	10.6	8.7	7.4	22.9	16.3	12.7	10.4	8.8	26.3	18.8	14.6	11.9	10.1	29.4	21.0	16.3	13.4	11.3	32.2	23.0	17.9	14.6	12.4
45	**250**	3.2	1.8	1.2	1.0	0.8	3.9	2.2	1.5	1.2	0.9	4.5	2.5	1.7	1.3	1.1	5.1	2.8	2.0	1.5	1.2	5.6	3.1	2.2	1.7	1.3

Appendix F – Most common values on parameters

Parameters					
Market size	α	434.3	Depreciation	δ	0.1
Price elasticity	β	-1.5	Part of adv.	μ	0.6
Cost of production	χ	10.0	Effect of sales	η	1.0
Cost of machine unit	σ	30.0	Number of firms	N	5
Price: Maximum	p^{MAX}	300.0	Minimum	p^{MIN}	10.01
Adv.: Maximum	a^{MAX}	425.0	Minimum	a^{MIN}	0
Check. Acc.: Start	C_1	200.0	Minimum	C^{MIN}	-1000.0

Appendix G – Summary of the model

Variables			***Test period*** ***Period 1***	***Period 2***
Capacity	(1)	K	0	10.8
Stocks	(2)	S	0	0
Check. acc.	(3)	C	200	-92.20
Cumalative adv.	(4)	A	0	72.00
Equity	(5)	$E = \sigma K + \chi S + C$	200	231.80
Investment	(6)	i	12	1.2
Production	(7)	o	12	12
Advertising	(8)	a	100	100
Price	(9)	p	30	30
Capacity	(10)	$\dot{K} = K + i$	12	12
Supply	(11)	$\dot{S} = S + o$	12	12
Payments	(12)	$P = \sigma i + \chi o + a$	580	256
Check account	(13)	$\dot{C} = C - P$	-380	348.2
Cumulative adv.	(14)	$\dot{A} = A + a$	100	172
Interest rate	(15)	r	0.19	0.18
Demand	(25)	$d(p, \dot{A}, p_n, \dot{A}_n)$	12.6	14.2
Sales	(26)	$q = \min[d, \dot{S}]$	12.0	12.0
Capacity	(28)	$\ddot{K} = (1-\delta)\dot{K}$	10.8	10.8
Stocks	(29)	$\ddot{S} = \dot{S} - q$	0	0
Check. acc.	(30)	$\ddot{C} = \dot{C} + pq + r\dot{C}$	-92.20	-50.88
Cumulative adv.	(31)	$\ddot{A} = \mu\dot{A} + \eta q$	72.00	115.20
Equity	(32)	$\ddot{E} = \ddot{C} + \sigma\ddot{K} + \chi\ddot{S}$	231.80	273.12
Profit	(34)	$\Pi = \ddot{E} - E$	31.80	41.32

The two columns to the right contain examples of decisions and results in the first two periods that are typical, but not optimal.

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