The Abnormal Earnings Growth Model: Applicability and Applications

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Abstract

We investigate a disaggregated version of the abnormal earnings growth (AEG) model of Ohlson and Juettner-Nauroth (2005). The value of the firm then becomes discounted free cash flows minus initial debt. Discounted free cash flows are equal to capitalized operating earnings from the initial stock of operating assets plus the present value of an infinite sequence of growth projects, where each growth project is valued by discounted economic value added. Sufficient conditions for the present value of the free cash flows to be equal to the sum of these two components are investigated. The Gordon growth formula is found to be one special case. Another case concerns lumpy growth projects with depreciation according to the annuity method. We then allow for three different interest rates, the required rate of return on equity under all-equity financing, the borrowing rate, and the required rate of return on equity under partial debt financing (the latter given by MM's Proposition 2). In the model of Ohlson and Juettner-Nauroth, these rates are the same. A firm-level model is developed that focuses on operating earnings and free cash flows with discounting at the required rate of return under all-equity financing. An equity-level model is then developed that focuses on bottom-line earnings and dividends with discounting at the required rate of return under partial debt financing. Relationships between the two models are explored. Dividend policy irrelevance holds only in a limited sense for the equity-level model.

Keywords: Financial analysis, abnormal earnings growth model, dividend policy, discounted dividends, discounted free cash flows, capitalized earnings, discounted economic value added.

JEL classification codes: G31, M49

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1 Introduction and Overview

Research, teaching, and also practice in the area of financial analysis and valuation have been affected over the last years by the abnormal earnings growth (AEG) valuation model, proposed in Ohlson and Juettner-Nauroth 2005 (from now on referred to as OJ 2005). The AEG model focuses on future earnings and earnings growth. In particular, a parsimonious version based on next year expected earnings and expected abnormal earnings growth in the short and the long run is promoted in OJ 2005. As the authors put it: "In a very real sense, the core of the [AEG] model shows how the current price depends on forward eps and their subsequent growth as captured by two dividend-policy independent measures of eps growth" (OJ 2005, p. 350). This focus on the prediction of earnings, rather than on the future distribution of wealth (i. e., dividends net of capital contributions), is an attractive model feature. More precisely, there is dividend policy irrelevance, meaning that the value of owners' equity does not depend on parameters relating to the choice of a particular dividend policy.

Even though the AEG model is attractive, it has the weakness that there is only one interest rate. In other words, the required unlevered rate of return on the equity, the borrowing rate, and the required rate of return on the equity under partial debt financing are all the same. The original AEG model is rather general. It is based on earnings and dividends in a general sense, without consideration of their components. A more detailed model that specifies bottom-line earnings as operating earnings minus debt interest, and dividend as free cash flow minus interest on debt plus debt increase, brings out the need for more than one interest rate.

This paper proceeds in two steps. First, we keep the assumption of one interest rate but proceed with the more detailed model. The value of the firm, obtained by discounting expected dividends, then becomes the present value of free cash flows minus intial debt. Interestingly, the present value of free cash flows is equal to capitalized operating earnings from an initial stock of operating assets plus the present value of a sequence of growth projects, where the size of each project increases at a given growth rate. Each growth project generates a level stream of economic value added. The value of each project can thus be determined by discounting its economic value added. This raises the question: Under what conditions can the value of the firm's operations, i. e., the present value of the free cash flows, be obtained as capitalized initial operating earnings plus the present value of an infinite sequence of growth projects, with each project valued by discounting its economic value added? The Gordon growth formula turns out to be one special case. There is also another important class of growth projects, more realistic than the Gordon formula, that can be handled by the AEG model: Projects where the initial investment in property, plant and equipment (PPE) is made periodically and where depreciation is according to the annuity method. A project of this type generates a level stream of economic value added. Hence, the conclusion is that the AEG model is applicable to a wider set of steady-state situations than the Gordon formula.

In our second step, we distinguish explicitly between the required rate of return on the equity under all-equity financing, the borrowing rate, and the required rate of return on the equity under partial debt financing. One can then develop both firm-level (focused on operating earnings and free cash flows) and equity-level (focused on bottom-line earnings and dividends) AEG models. The original AEG model can thus be generalized to a situation where the cost of equity capital is different from the debt rate. In a comparison between the firm-level and equity-level models, we find the former appealing. One possible application of the former is for continuing value in firm valuation models such as the discounted cash flow model.

Our analysis is conditioned on the following assumptions. First, financial borrowing and/or lending is used to obtain variations in the dividend policy. For simplicity, from now on we will say "borrowing" and "debt", even though these two could be negative, i. e., lending and financial assets. Second, since there is borrowing, there is a need for an explicit borrowing rate that is different from the discount rate. In Section 3, we follow OJ 2005 and Ohlson and Gao 2006 (from now on abbreviated as OG 2006) in letting the borrowing rate be the same as the discount rate. Starting in Section 5, however, there are two exogenous interest rates, the required rate of return on the unlevered operating activities ρ_u, and r for borrowing. Investors are assumed risk averse in the sense that $\rho_u > r$. Third, there are no company taxes. This implies that the leverage irrelevance proposition of Modigliani and Miller (1958) should hold. Consequently, the required rate of return under partial debt financing is determined in accordance with these authors' Proposition 2. Fourth, the clean surplus relationship holds (this assumption is not imposed in OJ 2005). This implies that a year's dividend is equal to free cash flow minus interest on debt plus increase in debt. For completeness, we also point out that by assumption the value of the equity is the present value of discounted expected dividends.

This paper is organized as follows. In the next section, we summarize the AEG model and the meaning of dividend policy irrelevance. Section 3 considers the detailed AEG model with one single interest rate. Sufficient conditions for the AEG model to be valid are investigated in Section 4. In Section 5, we introduce the required rate of return $\rho_u > r$ on the unlevered operating activities and generalize the model from Section 3 to the firm-level model with two exogenous interest rates ρ_u and r. The corresponding equity-level model is developed in Section 6. Section 7 contains concluding remarks. A separate appendix contains the details of some discounting operations.

2 The AEG Model and Dividend Policy Irrelevance

The AEG model is written as follows (cf. OJ 2005, p. 352):

$$P_1 = \frac{\text{eps}_1}{r} + \sum_{t=1}^{\infty}(1+r)^{-t}z_t \qquad \text{where} \quad z_t = \frac{1}{r}(\text{eps}_{t+1} + r\text{dps}_t - (1+r)\text{eps}_t). \tag{1}$$

Equation (1) provides the share value P_1 at the beginning of year 1, the date of valuation. eps_t means expected bottom-line earnings per share at the end of year t and dps_t the expected dividend at the same date. z_t is the capitalized increase in abnormal earnings per share between year t and year $t+1$. All variables referring to future dates should be interpreted as expected values, but for simplicity we suppress expectation operators. The discount rate is r. Equation (1) is a simple restatement of the fact that the value of one share is equal to the present value of future expected dividends. The interpretation of (1) is straight-forward: The value of one share is equal to capitalized year 1 earnings per share (the first term) plus additional value due to abnormal cum-dividend earnings per share growth (the second term) (cf. Penman 2007, pp. 204-206). It is stated in OG 2006 (p. 11) that z_t should be viewed as a function of eps_{t+1} and eps_t, even though it is also a function of dps_t. This is the non-parsimonious AEG model.

It is claimed in OJ 2005 (p. 353) that the following assumption "virtually suggests itself": $z_{t+1} = (1+g)z_t$, $0 \le g < r$. g is apparently a growth factor. When this assumption is made, the parsimonious AEG model is obtained. The discussion in OJ 2005 and in OG 2006 is mainly concerned with the parsimonious AEG model, as will be this paper.

The assumption that z_t grows by a constant factor g implies a crucial change in model logic: z_t is no longer a function of eps_{t+1} and eps_t. Instead, it is eps_{t+1} that gets determined by eps_t, z_t, and dps_t. The per share value is now:

$$P_1 = \frac{\text{eps}_1}{r} + \frac{z_1}{r-g}. \tag{2}$$

Certain insights can be obtained from a somewhat more circumstantial path to the per share value. For simplicity like in OG 2006 (p. 9), it is assumed that there is only one share outstanding, so all variables refer to the whole firm. Changing to notation that we will use from now on (rather similar to OG 2006), the AEG model can be written as a difference equation system

$$\begin{cases} x_{t+1} &= (1+r)x_t + rz_t - rd_t \\ z_{t+1} &= (1+g)z_t \\ d_{t+1} &= c_1x_t + c_2z_t + c_3d_t \end{cases} \tag{3}$$

with initial (expected) values x_1, z_1, and d_1. x_t denotes bottom-line earnings at the end of year t and d_t the dividend at the end of the same year. The first equation in (3) follows from the definition of z_t. c_1, c_2, and c_3 are arbitrary dividend policy parameters.[1]

The first equation in (3) shows clearly the dependence of x_{t+1} on x_t, z_t and d_t. Since z_t is the *capitalized* increase in abnormal bottom-line earnings between year t and year $t+1$, rz_t is the (non-capitalized) abnormal bottom-line earnings increase between those two years. One can interpret the first equation as follows. Suppose that all earnings are cash earnings. The firm has two sources of cash earnings. The first one is a bank account with an initial balance of x_1/r (so r is the bank rate). The second one is an autonomous project generator. The first project is initiated at the end of year 1 and provides a level stream of cash earnings equal to rz_1 at the end of each year, starting in year 2. The second project is initiated at the end of year 2 and provides a level stream of cash earnings equal to $rz_1(1+g)$ at the end of each year, starting in year 3, and so on. Total cash earnings from these projects are hence rz_1 at the end of year 2, $rz_1 + rz_1(1+g)$ at the end of year 3, $rz_1 + rz_1(1+g) + rz_1(1+g)^2$ at the end of year 4, etc. At the end of year $t+1$, they are $rz_1 \sum_{s=0}^{t-1}(1+g)^s = rz_1 \frac{1-(1+g)^t}{-g}$, assuming $g \neq 0$.[2] The earnings from the autonomous project generator are abnormal, since there are no initial investments associated with each successive project.

The first equation apparently says that year $t+1$ bottom-line cash earnings (x_{t+1}) are equal to year t bottom-line cash earnings (x_t) plus the increase in abnormal cash earnings between years t and $t+1$ (rz_t) plus interest on year t retained bottom-line cash earnings ($r(x_t - d_t)$). However, a different decomposition of x_{t+1}, suggested in the previous paragraph, is instructive: x_{t+1} is equal to the initial cash earnings x_1 plus abnormal cash earnings from the autonomous project generator $rz_1 \frac{1-(1+g)^t}{-g}$ plus interest on cumulated retained bottom-line cash earnings $r\sum_{s=1}^{t}(x_t - d_t)$. Supposedly cash earnings can be transferred at the interest rate r from one year to another by means of increasing or decreasing the debt (or withdrawing from or adding to the bank account), or equivalently, by means of the dividend policy. Such transfers do not create or destroy value. The value of the equity is hence the sum of the discounted values of the first two components:

$$E_1 = \frac{x_1}{r} + rz_1 \frac{1}{1+r} \sum_{t=1}^{\infty} \frac{\sum_{s=0}^{t-1}(1+g)^s}{(1+r)^t} = \frac{x_1}{r} + rz_1 \frac{1}{1+r} \sum_{t=1}^{\infty} \frac{\frac{1-(1+g)^t}{-g}}{(1+r)^t} = \frac{x_1}{r} + \frac{z_1}{r-g}. \quad (4)$$

E_1 denotes the computed value of the equity at the beginning of year 1, i. e., the time of valuation.[3] This is of course the same equity value as in (2). The previous interpretation

[1]The only restriction on these parameters is that c_1 and c_3 are such that the summation in equation (23) in the appendix converges (c_2 does not matter). This restriction holds for reasonable c_1 and c_3 values.

[2]If $g = 0$, then $rz_1 \sum_{s=0}^{t-1}(1+g)^s = rz_1(t-1)$. For simplicity, this case is disregarded in subsequent equations.

[3]Hence E_1 corresponds to P_1 above. More generally, E_t denotes computed value of the equity at the start of year t according to the valuation model.

of the AEG model is apparently quite close to the bank account metaphor that is often used by Ohlson (cf. Christensen and Feltham 2003, p. 287).

We know the value of the equity from (2) or (4), but the same equity value can obviously be obtained through direct discounting of expected dividends; cf. the appendix. It is noted that there are only four parameters in the valuation formula (4): bottom-line earnings at the end of year 1 (x_1), capitalized abnormal earnings growth between year 1 and year 2 (z_1), long-term growth g, and the discount rate r. In particular, E_1 does not depend on c_1, c_2, and c_3. Also, it does not depend on d_1.[4] Dividend policy irrelevance is defined by this independence of E_1 on c_1, c_2, c_3, and d_1.

3 The AEG Model with Operating and Financial Activities

The AEG model as outlined in Section 2 is not very specific about earnings. Most of the discussion in OJ 2005 and in OG 2006 is merely about "earnings" in an unspecified sense.[5] Also, the discount rate r is simply considered as an unexplained and exogenous cost of equity capital.

From now on, we distinguish between operating and bottom-line earnings. The difference between the two is interest expense on debt. Hence, there is a need for a debt rate that is typically not the same as the one that is used for discounting expected dividends to a present value (cf. OG 2006 pp. 29 and 49 for brief comments in two footnotes). In this section, however, we will nevertheless stick to the original model framework with only one interest rate. This means that the firm's operating and financing activities must be thought of as belonging to the same risk class, or that investors are risk-neutral and hence require the same cost of capital for risky and risk-free investments.

The variables are now ox_t (operating earnings), z_t (still assumed to grow at the rate g from year to year), f_t (free cash flow, assumed equal to $k \cdot ox_t$, where $0 < k \leq 1$), D_t (debt), and d_t (dividend). All of these variables except D_t are flow variables at the end of year t. D_t is a stock variable at the beginning of year t. It is assumed that D_t represents a book value that is equal to market value. The current point in time is the beginning of year 1, so all variables except D_1 are expected values. The required rate of return on the equity under all-equity financing and the borrowing rate are the same, denoted by r. Bottom-line earnings are $ox_t - rD_t$. By the equality of free cash flow and financial cash flow, and since the clean surplus relationship is assumed to hold, the dividend d_t is

[4] It holds that $\frac{\partial E_1}{\partial d_1} = 0$, since z_1 is a given initial value.

[5] Exceptions to this statement are Appendix II of OJ 2005 and pp. 27-30, 61-64, and 69-71 of OG 2006.

$f_t - rD_t + (D_{t+1} - D_t)$.[6]

From the definition of z_t (cf. (1)), it follows:

$$rz_t = (ox_{t+1} - rD_{t+1}) + r(f_t - rD_t + (D_{t+1} - D_t)) - (1+r)(ox_t - rD_t), \tag{5}$$

which may be simplified to

$$rz_t = ox_{t+1} + rf_t - (1+r)ox_t. \tag{6}$$

In other words, the debt terms cancel in (5). The difference equation system for this version of the AEG model can be written as follows, using (6) for the first equation:

$$\begin{cases} ox_{t+1} &= (1+r)ox_t + rz_t - rf_t \\ z_{t+1} &= (1+g)z_t \\ f_{t+1} &= k(1+r)ox_t + krz_t - krf_t \\ D_{t+1} &= \alpha_1 ox_t + \alpha_2 z_t + \alpha_3 f_t + \alpha_4 D_t + \alpha_5 d_t \\ d_{t+1} &= f_{t+1} - rD_{t+1} + (D_{t+2} - D_{t+1}) \end{cases} \tag{7}$$

In the fourth equation of (7), the firm's debt policy and hence also the dividend policy is specified by the parameters α_1, α_2, α_3, α_4, and α_5. ox_1, z_1, $f_1 = k \cdot ox_1$, D_1, and $d_1 = (f_1 - (1+r)D_1 + \alpha_1 ox_1 + \alpha_2 z_1 + \alpha_3 f_1 + \alpha_4 D_1)/(1 - \alpha_5)$ are given initial values.

An economic interpretation of the first equation in (7) is as follows. There are two exogenous sources of operating earnings, an original, already existing (at the start of year 1) stock of operating assets that provides a level stream of annual operating earnings equal to ox_1 at the end of each year, plus a generator of new projects. The first new project is undertaken at the end of year 1 and provides a level stream of annual abnormal operating earnings, i. e., economic value added, equal to rz_1 starting at the end of year 2. The second new project is undertaken at the end of year 2 and provides a level stream of annual abnormal operating earnings equal to $rz_1(1+g) = rz_2$ starting at the end of year 3, etc. These new projects will be referred to as z projects.

[6] The assumption that $f_t = k \cdot ox_t$ should not be interpreted to mean that the precise portion $(1-k)$ of operating earnings ox_t is needed for capital expenditures greater than depreciation and for additional working capital. Rather, k expresses that fraction of operating earnings that the firm freely chooses to pay out in the form of dividend and debt service. More precisely, and as will be seen in what follows, the firm is committed to a geometrically increasing series of growth projects, referred to as z projects. The initial growth project investment in year t is included in that portion of gross cash flow (operating earnings plus depreciation) that is not paid out as dividend and debt service. However, that portion of gross cash flow also comprises the initial investment in a zero-NPV operating project. The size of the latter can be freely chosen by the firm, and in that sense k is a parameter at the firm's discretion. In Appendix II of OJ 2005, a more complex formulation is used, $f_t = [q_1 - q_2(4+\sqrt{t})^{-1}]ox_t$ with $q_1 = 0.6$, $q_2 = 0.3$. Our simpler assumption $f_t = k \cdot ox_t$ permits us to obtain an explicit solution to the first three equations of the equation system (7) (cf. the appendix).

The present value of an ongoing project is the book value of operating net assets plus the discounted value of economic value added. In the net present value just prior to starting up the project, the book value of the operating net assets is cancelled by the initial investment. This means that the value of the first z project (at the end of year 1, prior to making the initial investment) is $(rz_1)/r = z_1$. Similarly, the net present value of the second z project (at the end of year 2) is $(rz_2)/r = z_2 = z_1(1+g)$, etc.

The first equation in (7) hence means that operating earnings in year $t+1$ (ox_{t+1}) are equal to operating earnings in year t (ox_t) plus the increase in abnormal operating earnings between years t and $t+1$ (rz_t) plus the (normal) return on that portion of the operating earnings in year t that was not distributed to the capital owners (debt and equity owners) in the form of financial cash flow ($r(ox_t - f_t) = r(1-k)ox_t$, since $f_t = k \cdot ox_t$). The non-abnormal part of the operating earnings increase in year $t+1$ due to the z project that is undertaken at the end of year t is included in the (normal) return on that part of the operating earnings in year t that is not distributed as financial cash flow.

Apparently, according to the first equation in (7), it is possible to reshuffle operating earnings from one year to another at the interest rate r. So there is actually the following implicit assumption: The firm has at its disposal a sufficiently large set of *zero-NPV* operating investment projects that can be utilized for such reshuffling. These zero-NPV projects, as well as the initial investments in z projects, are financed out of gross cash flows (or create additional gross cash flows). Similarly to the alternative decomposition of x_{t+1} in Section 2, there is the following alternative decomposition of ox_{t+1}: (i) The original operating earnings ox_1, plus (ii) abnormal operating earnings from the (cumulated set of) z projects, plus (iii) normal operating earnings on the z projects, plus (iv) operating earnings on the (cumulated set of) zero-NPV projects. (Cf. also equation (20) in Section 6 below.)

Rewriting the last equation of (7) in order to obtain variables pertaining to year t on the right hand side, one obtains:

$$\begin{aligned} d_{t+1} &= [f_{t+1} - (1+r)D_{t+1} + \alpha_1 ox_{t+1} + \alpha_2 z_{t+1} + \alpha_3 f_{t+1} + \alpha_4 D_{t+1}]/(1-\alpha_5) \\ &= \frac{(1+\alpha_3)(1+r)k + \alpha_1\alpha_4}{1-\alpha_5} ox_t + \frac{\alpha_1 r + \alpha_2(1+g) + (1+\alpha_3)rk - (1+r-\alpha_4)\alpha_2}{1-\alpha_5} z_t \\ &- \frac{\alpha_1 r + (1+\alpha_3)rk + (1+r-\alpha_4)\alpha_3}{1-\alpha_5} f_t - \frac{(1+r-\alpha_4)\alpha_4}{1-\alpha_5} D_t - \frac{(1+r-\alpha_4)\alpha_5}{1-\alpha_5} d_t. \end{aligned} \tag{8}$$

Discounting expected dividends to a present value (at the interest rate r), one obtains (see the appendix)

$$E_1 = \frac{ox_1}{r} + \frac{z_1}{r-g} - D_1.$$

The value of the equity is *discounted free cash flows minus initial debt.* This follows since the first two terms are the present value of future free cash flows (i. e., the value of the operations), as can be shown by direct calculation (again see the appendix).

The present value of future free cash flows in the AEG model is apparently *independent of the free cash flows*, so there is free cash flow irrelevance. This follows since the parameter k that relates free cash flows to operating earnings does not enter into the formula for the present value of the free cash flows. The implication is that the present value of the free cash flows is the same when each year's free cash flow is (say) 80% of the year's operating earnings as when it is (say) 50% of the year's operating earnings. At first sight, this may seem puzzling. However, free cash flow irrelevance is quite logical in the AEG model, if one remembers that the parameter k reshuffles operating earnings between years by means of zero-NPV operating investment projects. Free cash flow irrelevance is not an assumption that we have imposed. It is inherent, although not immediately visible, in the AEG model. Ohlson and Juettner-Nauroth recognize this when they say "... the model builds in free cash flows irrelevancy no less than dividend policy irrelevancy" (OJ 2005, p. 363).

To be pedantic, free cash flow irrelevance does not hold if ox_1 or z_1 depend on k. One special subcase of this kind is mentioned in the appendix, i. e., when $z_1 = -ox_1\{(r(1-k)-g)/r\}$. In that situation, the present value of the free cash flows is $f_1/(r-g) = k \cdot ox_1/(r-g)$, contradicting free cash flow irrelevance.

4 Conditions for the AEG Model to Hold

The value of the firm's operations, i. e., the present value of free cash flows, actually follows directly from the previously suggested decomposition of ox_{t+1} into four components and can be paraphrased as follows: Capitalized level operating earnings from the firm's initial stock of operating assets (ox_1/r), plus the present value of an infinite, geometrically increasing sequence of z projects ($\sum_{t=1}^{\infty} z_t/(1+r)^t = \sum_{t=1}^{\infty}(z_1(1+g)^{t-1})/(1+r)^t = z_1/(r-g)$). z_t is the net present value of the t-th z project, with initial investment at the end of year t. Each z project provides a level stream of economic value added equal to rz_t, so its net present value at the time it is initiated is $(rz_t)/r = z_t$. The last two components of the suggested decomposition, normal operating earnings on z projects and operating earnings on zero-NPV projects, do not provide or destroy any value and hence do not enter into the present value of the free cash flows.

The question is now: Under what conditions is the present value of the firm's free cash flows equal to $ox_1/r + z_1/(r-g)$? To begin with, the firm must be in a steady state. This follows from the infinite nature of the capitalization and discounting operations. But there are also rather specific conditions on earnings patterns and associated accounting rules.

To get some feeling for those conditions, we first consider the Gordon growth formula.

The Gordon formula is very often used in the discounted cash flow model for firm valuation to provide the continuing value at the outset of the post-horizon period (cf. Jennergren 2007). More precisely, it is assumed that the free cash flows f_t in the post-horizon period increase year by year by some growth rate g. Suppose year 1 is the first year of the post-horizon period. The value of the operations is then $f_1/(r-g)$. Since for the time being we do not distinguish between different discount rates, r is still the rate that is valid for discounting the free cash flows from the operations.

Table 1 contains a numerical example of the situation that must hold when the Gordon formula is applied. Columns [3], [5], [6], [8], [12], and [14] contain stock variables at year starts, the remaining columns flow variables at year ends.[7] Given initial values in columns [1], [2], and [8] are indicated by underlining. The current year is year 1, and the object of the valuation is the value of the operations at the beginning of that year. The economic life of the PPE is 10 years, and annual depreciation is linear, i. e., 1/10 of the acquisition value of each cohort.[8] The history of the firm's development is hence given by years −10 to 0. The assumed growth rate g is 3.02% (for instance, due to 2% inflation and 1% real growth). Sales minus cash operating costs in column [1] and capital expenditures in column [2] have apparently increased by 3.02% between years −10 and 0 and are expected to grow at the same rate in future years. Gross PPE in column [3] is obtained by summing capital expenditures backwards over cohorts that have not been retired. For the beginning of year 1, it is apparently obtained as 10.3020 + 10.6131 + ... + 13.4653 = 118.2097. Accumulated depreciation in column [5] is also obtained by summing backwards over cohorts that have not been retired. For instance, for the beginning of year 1, accumulated depreciation is (9/10)· 10.3020 + (8/10)·10.6131 + ... + (0/10)·13.4653 = 50.2971. Net PPE in column [6] is gross PPE minus accumulated depreciation. Depreciation in column [4] is equal to the year's starting gross PPE multiplied by 1/10. Each year's investment in working capital in column [7] is equal to the year's starting working capital in column [8] multiplied by the growth rate 3.02%.

The operating earnings ox_t in year t in column [9] are equal to sales minus cash operating costs in column [1] minus depreciation in column [4]. Free cash flow in column [10] is operating earnings in column [9] plus depreciation in column [4] minus capital expenditures in column [2] minus investment in working capital in column [7] and turns out to be 27.4087 in year 1. Suppose the discount rate r is 10%. The value of the

[7]Columns in Table 1 and Table 2 (later in this section) are marked by square brackets, to distinguish from equation numbers which are marked by parentheses.

[8]The Gordon growth formula being a special case of the AEG model does not depend on the depreciation rule. However, the value split between initial operations and growth projects changes, if the depreciation rule changes.

operations is then $27.4087/(0.1-0.0302) = 392.6751$.

However, there is an alternative interpretation of the Gordon formula. The firm's operations are viewed as consisting of two parts, the first one being the original operations that provide operating earnings equal to ox_1 in every single year (column [11]). In addition, the firm has an infinite sequence of growth projects. In connection with the Gordon formula, those growth projects are also referred to as Gordon projects. The first project is undertaken at the end of year 1 and provides an infinite, level stream of operating earnings equal to $g \cdot ox_1$, starting in year 2. The second project is undertaken at the end of year 2 and provides an infinite, level stream of operating earnings equal to $g \cdot ox_1(1+g)$ starting in year 3, etc. The sum of these infinite, level streams from the growth projects is given in column [13].

The investment in PPE that is necessary for each growth project is equal to capital expenditures in column [2] minus depreciation in column [4]. For year 1, the investment in PPE for the first growth project is equal to $13.8719 - 11.8210 = 2.0510$ (after Excel rounding), which is also equal to the increase in net PPE $0.0302 \cdot 67.9126$ between the start of year 1 and the start of year 2. Out of the total capital expenditures at the end of year 1 of 13.8719, 11.8210 hence corresponds to maintaining the initial PPE that the firm has already at the start of year 1 and that is necessary for sustaining the original operating earnings of ox_1. Net PPE of 2.0510 (all age cohorts) is transferred to the first growth project, so the initial investment in PPE for that project is 2.0510. The stocks of PPE associated with both operating earnings streams (ox_1 and the first growth project stream) have *the same age structures*. That is, the acquisition values of successive non-retired cohorts increase in line with the assumed growth rate g.[9] It is this geometrically increasing age structure that guarantees that depreciation for the PPE associated with the original stream ox_1 is equal to capital expenditures for that stock of PPE. This observation generalizes to later years t: Depreciation of PPE related to the original stream ox_1 and to all previously initiated growth projects is equal to capital expenditures, for each associated stock of PPE considered separately.

Adding investment in PPE and investment in working capital, one obtains each year's total new investment (associated with that year's growth project). Since depreciation equals capital expenditures for growth projects that are already up and running, the book value of operating net assets is always equal to cumulated total new investments for these projects, and is given in column [14] (equal to column [2] minus column [4] plus column [7] plus last year's value in column [14]). Let O_t denote the book value of total

[9] The gross PPE that is associated with the original operating earnings at the start of year 2 is hence $(1-0.0302) \cdot (10.6131 + 10.9336 + ... + 13.8719) = 118.2097$, and the gross PPE that is associated with the first growth project at the start of year 2 is $0.0302 \cdot (10.6131 + 10.9336 + ... + 13.8719) = 121.7797 - 118.2097 = 3.5699$. The corresponding net PPEs are $(1-0.0302) \cdot ((1/10) \cdot 10.6131 + (2/10) \cdot 10.9336 + ... + (10/10) \cdot 13.8719) = 67.9126$ and $0.0302 \cdot ((1/10) \cdot 10.6131 + (2/10) \cdot 10.9336 + ... + (10/10) \cdot 13.8719) = 2.0510$.

operating net assets (net PPE plus working capital) at the start of year t. The book value of new investment that is associated with the growth project in year t is apparently $gO_1(1+g)^{t-1}$.

The firm is hence expected to generate the perpetual, level operating earnings stream ox_1 associated with the operating assets available at the start of year 1 and the perpetual, level operating earnings streams $g \cdot ox_1(1+g)^{t-1}$ ($t = 1, 2, 3, ...$) that are associated with the growth projects. These perpetual, level operating earnings streams are *cash streams* (i. e., free cash flows), since depreciation of the PPE that is associated with each stream exactly matches capital expenditures necessary for maintaining that net PPE, and since no additional working capital is necessary (since the streams are level).

The net present value of the growth project that is started in year t is evidently $(g \cdot ox_1(1+g)^{t-1})/r - gO_1(1+g)^{t-1} = [g \cdot ox_1(1+g)^{t-1} - rgO_1(1+g)^{t-1}]/r$. The second way of writing this net present value is recognized as discounted economic value added and corresponds to z_t in the AEG model. Summing the capitalized value of the initial operating earnings stream ox_1 and the net present values of all growth projects, the value of the firm's operations is obtained as

$$\frac{ox_1}{r} + \frac{z_1}{r-g} = \frac{ox_1}{r} + \frac{1}{r-g} \times \frac{g \cdot ox_1 - rgO_1}{r} = \frac{ox_1 - gO_1}{r-g},$$

which is of course equal to the Gordon formula since $ox_1 - gO_1$ is free cash flow f_1. Apparently, the Gordon formula can be interpreted as capitalized initial operating earnings plus discounted economic value added of all growth projects. In the example in Table 1, the value of the operations can thus also be written as

$$\frac{29.7948}{0.1} + \frac{1}{0.1 - 0.0302} \times \frac{0.0302 \cdot 29.7948 - 0.1 \cdot 0.0302 \cdot (67.9126 + 11.0975)}{0.1}$$
$$= \frac{29.7948}{0.1} + \frac{1}{0.1 - 0.0302} \times \frac{0.8998 - 0.1 \cdot 2.3861}{0.1} = 392.6751.$$

Hence, the Gordon formula is a special case of the AEG model, with the Gordon projects corresponding to more general z projects in the AEG model. The accounting for PPE is rather peculiar when the Gordon formula is interpreted as a special case of the AEG model: The stock of PPE that is associated with each Gordon project is a slice of the total stock of PPE for the entire firm, age cohort by age cohort.

There is another important class of growth projects that can be incorporated into the AEG model: Projects where the initial investment in PPE consists of one piece of brand new equipment (i. e., not including all non-retired age cohorts), and where depreciation is according to the annuity method. Such projects will be referred to here as lumpy projects. To remind the reader of depreciation by the annuity method, suppose an initial investment of 1 is to be depreciated over its economic life n years. Depreciation at the end of year i of the economic life (between 1 and n) is

$$1 \times \frac{r}{1-(1+r)^{-n}} - r \times 1 \times \frac{1-(1+r)^{i-n-1}}{1-(1+r)^{-n}} = \frac{r(1+r)^{i-n-1}}{1-(1+r)^{-n}},$$

where the first term on the left hand side is the total annuity (interest and depreciation) and the second term interest on the year's starting undepreciated value. If the initial investment in PPE is depreciated in this fashion, abnormal operating earnings are constant. Repeating the initial investment in PPE in an infinite chain, one hence obtains an infinite, level stream of abnormal operating earnings (the investment in working capital is only made once).

The example in Table 2 illustrates a single lumpy project, supposed to be the first in a sequence of such projects, where the scale of each project is $(1+g)$ times the predecessor. Columns [4], [5], and [6] contain stock variables at year starts, the remaining columns flow variables at year ends. The project is initiated at the end of year 1 with an initial investment in working capital of 6 and an initial investment in new PPE of 10. The PPE has an economic life of 5 years, so the PPE is replaced at the end of year 6, 11, etc. Depreciation according to the annuity method is shown in column [3]. Operating net assets is the sum of net PPE and working capital in columns [5] and [6]. Operating earnings in column [8] are sales minus cash operating costs in column [7] minus depreciation in column [3]. Economic value added in column [9] is operating earnings in column [8] minus operating net assets multiplied by the interest rate (assumed to be 10%). The important observation is that economic value added is the same, year after year over an infinite horizon. Operating earnings are not constant over time, and neither is free cash flow. It is the annuity depreciation method that provides the level stream of economic value added, so this particular choice of accounting principle is a precondition for the validity of the AEG model in situations where capital expenditures for a z project are made periodically, in big lumps.[10]

The present value of the project in Table 2 (at the start of year 2) is hence 0.0403/0.1. Suppose the growth rate g is 3.02%. The present value (at the start of year 1) of the infinite sequence of lumpy projects is then

$$\frac{1}{0.1-0.0302} \times \frac{0.0403}{0.1}.$$

The z projects in the AEG model can hence also be lumpy projects. If so, it is necessary that depreciation is according to the annuity method. If not, the stream of

[10]It is known in the literature that the annuity depreciation method results in constant economic value added, when a project's cash flows are level. Cf. Dutta and Reichelstein 2005, p. 549; Feltham and Ohlson 1996, p. 228.

economic value added will not be level, meaning that the AEG model cannot be applied. The z projects can obviously be combinations of Gordon projects and lumpy projects (with annuity depreciation), as long as the growth rate g is the same for both project categories.

At the end of this discussion of conditions that must hold for the (parsimonious) AEG model to be valid, we conclude that they are restrictive. The AEG model is only valid for steady-state settings. There are further restrictions on the accounting rules. The Gordon growth formula is a special case of the AEG model. However, the latter is a non-trivial extension of the former since lumpy projects can also be included in the z projects, if the annuity depreciation method is used.[11]

5 The Firm-Level AEG Model with Operating and Financial Activities and Three Different Interest Rates

Now let ρ_u denote the required rate of return on owners' equity under the assumption of all-equity financing, and let r be the borrowing rate. Consistent with MM's Proposition 2 (Modigliani and Miller 1958), the required rate of return on owners' equity under partial debt financing is

$$\rho_e = \rho_u + (\rho_u - r)\frac{D}{E}, \tag{9}$$

where D is the market value of the debt, assumed equal to its book value, and E the computed value of owners' equity (i. e., the value of the equity that results from the valuation model). These values of D and E are supposed to be valid at the beginning of that year to which (9) is applied. In the previous sections, it was assumed that $\rho_u = r$ and hence also $\rho_e = r$.

The difference equation system (7) in the Section 3 can now be generalized in the following manner to a situation with three different interest rates:

$$\begin{cases} ox_{t+1} &= (1+\rho_u)ox_t + \rho_u z_t - \rho_u f_t \\ z_{t+1} &= (1+g)z_t \\ f_{t+1} &= k(1+\rho_u)ox_t + k\rho_u z_t - k\rho_u f_t \\ D_{t+1} &= \alpha_1 ox_t + \alpha_2 z_t + \alpha_3 f_t + \alpha_4 D_t + \alpha_5 d_t \\ d_{t+1} &= f_{t+1} - rD_{t+1} + (D_{t+2} - D_{t+1}) \end{cases} \tag{10}$$

[11]The zero-NPV projects that are used to reshuffle operating earnings between years do not pose any particular requirements, i. e., any method of depreciation of PPE invested in such projects is permitted. The possibility of reshuffling is merely an obscuring aspect of the AEG model: It greatly complicates the difference equation structure without affecting the solution value.

ox_1, z_1, $f_1 = k \cdot ox_1$, D_1, and $d_1 = (f_1 - (1+r)D_1 + \alpha_1 ox_1 + \alpha_2 z_1 + \alpha_3 f_1 + \alpha_4 D_1)/(1 - \alpha_5)$ are given initial values. This equation system will be referred to as the *firm-level model*. It follows from the previous discussion that the value of the operations of the firm at the beginning of year t, i. e., the discounted value of the free cash flows, is $ox_t/\rho_u + z_t/(\rho_u - g)$.

The value E_1 of the equity at the beginning of year 1 can be derived by discounting expected dividends. A particular debt policy, and hence also dividend policy, is initially assumed: Until some horizon year T, the debt policy is arbitrary. However, at the beginning of year $T+1$, the debt D_{T+1} will be repaid through a new issue of owners' equity, and from then on there will be no debt at all ($D_t = 0$ for $t > T+1$). Under the horizon assumption, the value of owners' equity at the beginning of year $T+1$ is obviously

$$E_{T+1} = \frac{ox_{T+1}}{\rho_u} + \frac{z_{T+1}}{\rho_u - g} - D_{T+1}.$$

The equity value at the beginning of year T is then given by

$$E_T = \frac{f_T - rD_T - D_T + \frac{1}{\rho_u} ox_{T+1} + \frac{1}{\rho_u - g} z_{T+1}}{1 + \rho_u + (\rho_u - r)\frac{D_T}{E_T}} = \frac{\frac{1+\rho_u}{\rho_u} ox_T + \frac{1+\rho_u}{\rho_u - g} z_T - rD_T - D_T}{1 + \rho_u + (\rho_u - r)\frac{D_T}{E_T}},$$

where the repayment of debt at the beginning of year $T+1$ is accounted for in the dividend at the end of year T, and where the first, second, and fifth equations in (10) have been used. Solving for E_T, one obtains

$$E_T = \frac{ox_T}{\rho_u} + \frac{z_T}{\rho_u - g} - D_T.$$

Stepping backwards one year at a time,

$$E_1 = \frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1 \tag{11}$$

under the assumption that the debt D_{T+1} at the beginning of year $T+1$ is retired and that $D_t = 0$ for all $t > T+1$. Next, letting $T \to \infty$ we obtain (11) under a totally arbitrary dividend policy over an infinite number of years. More generally, $E_t = ox_t/\rho_u + z_t/(\rho_u - g) - D_t$. As in Section 3, the value of the equity is discounted free cash flows minus initial debt.

The implication is that there is free cash flow irrelevance and dividend policy irrelevance in the AEG model with three interest rates. This follows, since the free cash flow parameter k and the debt policy parameters α_1, α_2, α_3, α_4, and α_5 do not enter into the value formula (11).[12]

In the following section we compare bottom-line earnings as calculated from the firm-level model with bottom-line earnings in the equity-level model (cf. equations (20) and

[12]The non-appearance of d_1 in (11) no longer matters for dividend policy irrelevance, since d_1 is determined by α_1, α_2, α_3, α_4, α_5, k, ox_1, z_1, and D_1.

(21)). For that purpose only (i. e., without affecting the value of E_1), the capital structure in terms of D_t and E_t should be constant over time. This means that $D_{t+1}/(D_{t+1}+E_{t+1}) = D_{t+1}/(ox_{t+1}/\rho_u+z_{t+1}/(\rho_u-g))$ should be equal to $D_1/(D_1+E_1)$. In other words, it should hold

$$D_{t+1} = \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u-g}} \left(\frac{ox_{t+1}}{\rho_u} + \frac{z_{t+1}}{\rho_u - g}\right).$$

This means that α_1, α_2, α_3, α_4, and α_5 should be set as follows:

$$\alpha_1 = \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u-g}} \times \frac{1+\rho_u}{\rho_u},$$

$$\alpha_2 = \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u-g}} \times \frac{1+\rho_u}{\rho_u - g},$$

$$\alpha_3 = -\frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u-g}},$$

and $\alpha_4 = 0$, $\alpha_5 = 0$.

6 The Equity-Level AEG Model with Bottom-Line Earnings and the Required Return on Equity as Discount Rate

We now return to the original, non-detailed AEG model that focuses on bottom-line earnings. Define z_t^* as follows:

$$z_t^* = \frac{\rho_u(\rho_e - g)}{\rho_e(\rho_u - g)} z_t = \frac{\rho_u\left(\rho_u + (\rho_u - r)\frac{D_t}{E_t} - g\right)}{\left(\rho_u + (\rho_u - r)\frac{D_t}{E_t}\right)(\rho_u - g)} z_t$$

$$= \frac{\rho_u\left(\rho_u + (\rho_u - r)\frac{D_t}{\frac{ox_t}{\rho_u}+\frac{z_t}{\rho_u-g}-D_t} - g\right)}{\left(\rho_u + (\rho_u - r)\frac{D_t}{\frac{ox_t}{\rho_u}+\frac{z_t}{\rho_u-g}-D_t}\right)(\rho_u - g)} z_t \tag{12}$$

It is possible (although tedious) to show that

$$\frac{x_t}{\rho_e} + \frac{z_t^*}{\rho_e - g} = \frac{ox_t - rD_t}{\rho_u + (\rho_u - r)\frac{D_t}{E_t}} + \frac{\rho_u}{\rho_u + (\rho_u - r)\frac{D_t}{E_t}} \times \frac{z_t}{\rho_u - g}$$

$$= \frac{ox_t - rD_t}{\rho_u + (\rho_u - r)\frac{D_t}{\frac{ox_t}{\rho_u}+\frac{z_t}{\rho_u-g}-D_t}} + \frac{\rho_u}{\rho_u + (\rho_u - r)\frac{D_t}{\frac{ox_t}{\rho_u}+\frac{z_t}{\rho_u-g}-D_t}} \times \frac{z_t}{\rho_u - g}$$

$$= \frac{ox_t}{\rho_u} + \frac{z_t}{\rho_u - g} - D_t = E_t. \tag{13}$$

However, since we want to derive the value of owners' equity directly, not as a residual, we will set up an equation system corresponding to (3) in Section 2 above. We start as follows:

$$\begin{cases} x_{t+1} &= (1+\bar{\rho}_e)x_t + \bar{\rho}_e z_t^* - \bar{\rho}_e d_t \\ z_{t+1}^* &= (1+g)z_t^* \\ D_{t+1} &= \alpha_{31}x_t + \alpha_{32}z_t^* + \alpha_{33}D_t + \alpha_{34}d_t \\ d_{t+1} &= k(x_{t+1} + rD_{t+1}) - rD_{t+1} + (D_{t+2} - D_{t+1}) \end{cases} \tag{14}$$

This equation system will be referred to as the *equity-level model*. In (14), the equation for d_{t+1} uses $f_{t+1} = k \cdot ox_{t+1}$ and $ox_{t+1} = x_{t+1} + rD_{t+1}$. The debt policy parameters in the equation for D_{t+1} are denoted by two indices, to emphasize that these parameters are different from the corresponding parameters in the fourth equations of of (7) and (10).

The required rate of return on owners' equity $\bar{\rho}_e$ in (14) is constant, more precisely that rate $\bar{\rho}_e$ that is implied by the initial debt D_1. This means that $\bar{\rho}_e$ is given by

$$\bar{\rho}_e = \rho_u + (\rho_u - r)\frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1}. \tag{15}$$

In order to maintain the constant $\bar{\rho}_e$ according to (15), the debt levels D_{t+1} must change over time as follows:

$$D_{t+1} = \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1}\left(\frac{x_{t+1}}{\bar{\rho}_e} + \frac{z_{t+1}^*}{\bar{\rho}_e - g}\right) = \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1} \times \frac{1+\bar{\rho}_e}{\bar{\rho}_e}x_t$$

$$+\frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1} \times \frac{1+\bar{\rho}_e}{\bar{\rho}_e - g}z_t^* - \frac{D_1}{\frac{ox_1}{\rho_u} + \frac{z_1}{\rho_u - g} - D_1}d_t. \tag{16}$$

Since ox_1, z_1, and D_1 are given initial values, this equation identifies α_{31}, α_{32}, and α_{34} in the third equation of (14). Apparently, $\alpha_{33} = 0$. Collecting terms in the fourth equation of (14) and using the third equation of (14) with $\alpha_{33} = 0$,

$$d_{t+1} = kx_{t+1} - (1 + r(1-k))D_{t+1} + D_{t+2}$$

$$= \frac{1}{1-\alpha_{34}}\{(k+\alpha_{31})x_{t+1} + \alpha_{32}z_{t+1}^* - (1+r(1-k))D_{t+1}\}$$

$$= \frac{1}{1-\alpha_{34}}[(k+\alpha_{31})(1+\bar{\rho}_e) - (1+r(1-k))\alpha_{31}]x_t$$

$$+\frac{1}{1-\alpha_{34}}[(k+\alpha_{31})\bar{\rho}_e + \alpha_{32}(1+g) - (1+r(1-k))\alpha_{32}]z_t^*$$

$$-\frac{1}{1-\alpha_{34}}[(k+\alpha_{31})\bar{\rho}_e+(1+r(1-k))\alpha_{34}]d_t. \tag{17}$$

Our equation system then consists of the first two equations in the equity-level model (14), i. e., the equations for x_{t+1} and z^*_{t+1}, plus (17) as equation for d_{t+1}. The initial values are $x_1 = ox_1 - rD_1$, $z^*_1 = \frac{\rho_u(\bar{\rho}_e-g)}{\bar{\rho}_e(\rho_u-g)}z_1$, and $d_1 = \frac{1}{1-\alpha_{34}}\{(k+\alpha_{31})x_1+\alpha_{32}z^*_1-(1+r(1-k))D_1\}$.

Discounting expected dividends to a present value, as in Section 2, but at the required rate of return on the equity $\bar{\rho}_e$,

$$E_1 = \frac{x_1}{\bar{\rho}_e}+\frac{z^*_1}{\bar{\rho}_e-g}. \tag{18}$$

Apparently, this result is very similar to the one in Section 2, but with the required rate of return on owners' equity different from the borrowing rate. One can show (although again somewhat tediously) that successive values of x_{t+1}, as determined through the first equation of the equity-level model (14), are equal to $ox_{t+1} - rD_{t+1}$, where the ox_{t+1} are determined through the first equation of the firm-level model (10) and the D_{t+1} through (16).

Superficially, there is dividend policy irrelevance, since the value of the equity according to (18) seems independent of the debt policy parameters α_{31}, α_{32}, α_{33}, and α_{34}. Such a conclusion is simplistic, however, since the requirement for a constant ρ_e means that the parameters in the equation for d_{t+1} must be set as in (17). It is seen that α_{31}, α_{32}, and α_{34}, as well as the free cash flow parameter k, enter into those parameters. Moreover, α_{31}, α_{32}, and α_{34} have to be set as in (16). In fact, unless these conditions hold, the equity-level model is not even defined, since ρ_e is no longer constant.

However, by equation (13), it also follows that

$$E_1 = \frac{x_1}{\bar{\rho}_e}+\frac{z^*_1}{\bar{\rho}_e-g} = \frac{ox_1}{\rho_u}+\frac{z_1}{\rho_u-g}-D_1, \text{ where } z^*_1 = \frac{\rho_u(\bar{\rho}_e-g)}{\bar{\rho}_e(\rho_u-g)}z_1, \tag{19}$$

and where $\bar{\rho}_e$ is defined by (15). The right hand side of (19), the equity value according to the firm-level model, is totally independent of the dividend policy (and the free cash flow policy). The implication of (19) is that the equity value according to the equity-level model, $x_1/\bar{\rho}_e + z^*_1/(\bar{\rho}_e - g)$, is *valid for any dividend policy* that the firm contemplates starting in year 3, as will be seen presently. In other words, the equity-level model can be applied by *pretending* that the dividend (and debt) policy of the firm is the one that is implied by the constant required return on equity $\bar{\rho}_e$. The resulting equity value is the correct one, even if the actual dividend policy is a different one starting in year 3.

The significance of year 3 is the following. Suppose one wants to estimate z^*_1 from analysts' forecasts of x_1, x_2, and d_1, i. e., setting

$$z^*_1 = \frac{1}{\bar{\rho}_e}(x_2+\bar{\rho}_e d_1-(1+\bar{\rho}_e)x_1)$$

$$= \frac{1}{\bar{\rho}_e}((ox_2 - rD_2) + \bar{\rho}_e(k \cdot ox_1 - rD_1 + (D_2 - D_1)) - (1 + \bar{\rho}_e)(ox_1 - rD_1)).$$

The resulting estimated z_1^* is *not independent* of the debt policy. More precisely, it is not independent of D_2, and hence not of the dividend d_1 that is paid at the end of year 1. Only when the forecasts of x_2 and d_1 are consistent with $z_1^* = \frac{\rho_u(\bar{\rho}_e - g)}{\bar{\rho}_e(\rho_u - g)} z_1$, in particular with the constant leverage ratio in years 1 and 2 that is implied by $\bar{\rho}_e$, will the equity-level model generate the same value as the firm-level model. The present difficulty apparently does not arise, if $\rho_u = r$.

In summary, the equity-level model is only defined for one special debt policy where D_t/E_t is constant in all years. However, the resulting equity value is the correct one and holds for any other debt policy starting in year 3 as well. One may hence say that in a limited sense, dividend policy irrelevance holds in the equity-level model.

One can interpret the difference between $\bar{\rho}_e z_1^*$ in the equity-level model (14) and $\rho_u z_1$ in the firm-level model (10). Some additional notation is needed. As in Section 4, let O_t denote the book value of total operating net assets at the beginning of year t. O_1 is hence the book value of the original operating net assets at the beginning of year 1. In line with the interpretation of the Gordon formula in Section 4 above, depreciation of the original operating assets corresponds exactly to capital expenditures, and there is no need for additional working capital, so the book value of the operating net assets associated with the level stream of operating earnings ox_1 remains the same year after year.[13] O_{zt} denotes the book value of the operating net assets associated with the z projects at the start of year t, and O_{0t} the book value of the operating net assets associated with the zero-NPV projects also at the start of year t. Apparently, $O_{z1} = 0$ and $O_{01} = 0$. It then holds that $O_t = O_1 + O_{zt} + O_{0t}$. Let B_t denote the book equity at the start of year t. Hence, B_1 is the original book equity at the start of year 1 and $B_t - B_1$ the increase in book equity (equal to retained bottom-line earnings if there is no issue of shares) between the start of year 1 and the start of year t.

Assume that the debt policy parameters α_1, α_2, α_3, α_4, and α_5 in the firm-level model have been set as indicated at the end of the previous section. With α_{31}, α_{32}, α_{33}, and α_{34} in the equity-level model as indicated in the discussion following equation (16), bottom-line earnings according to the firm-level model must then be the same as bottom-line earnings in the equity-level model. Taking the components of operating earnings in the firm-level model, bottom-line earnings x_t at the end of year t can be written as

$$ox_1 + \rho_u z_1 \frac{1 - (1+g)^{t-1}}{-g} + \rho_u(O_{zt} + O_{0t}) - rD_1 - r(D_t - D_1)$$

[13] If the original operating assets pertain to a lumpy project situation, then it is possible to combine with zero-NPV projects in such a fashion that operating earnings from the original operating assets are constant over time, with the book value of the operating net assets also remaining constant over time. (This was not illustrated in Table 2 in Section 4.)

$$= ox_1 + \rho_u z_1 \frac{1-(1+g)^{t-1}}{-g} + \rho_u(O_t - O_1) - rD_1 - r(D_t - D_1). \quad (20)$$

Similarly, taking the components of bottom-line earnings in the equity-level model, using $x_1 = ox_1 - rD_1$, bottom-line earnings at the end of year t are also

$$ox_1 - rD_1 + \rho_u z_1 \frac{1-(1+g)^{t-1}}{-g} + (\bar{\rho}_e z_1^* - \rho_u z_1)\frac{1-(1+g)^{t-1}}{-g} + \bar{\rho}_e(B_t - B_1). \quad (21)$$

The last term in (21) is the return on retained bottom-line earnings. It is an implicit assumption in the firm-level AEG model that bottom-line earnings can be reinvested at the discounting rate, i. e., $\bar{\rho}_e$. In other words, retained bottom-line earnings are related to zero-NPV projects in the equity-level model, just as retained operating earnings are related to zero-NPV projects in the firm-level model.

Setting (20) equal to (21),

$$(\bar{\rho}_e z_1^* - \rho_u z_1)\frac{1-(1+g)^{t-1}}{-g} = \rho_u(O_t - O_1) - \bar{\rho}_e(B_t - B_1) - r(D_t - D_1).$$

Since $(B_t - B_1) = (O_t - O_1) - (D_t - D_1)$, it follows that

$$(\bar{\rho}_e z_1^* - \rho_u z_1)\frac{1-(1+g)^{t-1}}{-g} = -(\bar{\rho}_e - \rho_u)(O_t - O_1) + (\bar{\rho}_e - r)(D_t - D_1). \quad (22)$$

The difference between the earnings streams from the z projects in the firm-level model and the z^* projects in the equity-level model hence has to do with an inconsistency in the way in which returns to the equity holders from retained bottom-line earnings (returns on zero-NPV projects plus normal returns on z projects) are evaluated in the two models. In the firm-level model, those returns are (correctly) evaluated as $\rho_u(O_t - O_1) - r(D_t - D_1)$, whereas in the equity-level model they are evaluated as $\bar{\rho}_e(B_t - B_1) = \bar{\rho}_e(O_t - O_1) - \bar{\rho}_e(D_t - D_1)$. This inconsistency is picked up in the difference between z_t and z_t^*.

7 Conclusion

The starting point in this paper was the observation that earnings are rather general, or generic, in the AEG model in OJ 2005 and OG 2006. Disregarding the need for more than one interest rate, Section 3 reformulated the original AEG model to focus on operating earnings and free cash flows rather than on earnings and dividends. Even though dividends are discounted, the equity value turns out to be discounted free cash flows minus initial debt. Moreover, the value of the free cash flows is equal to capitalized year 1 operating earnings associated with the firm's initial stock of operating assets plus the present value of abnormal operating earnings from a geometrically increasing sequence of growth projects (also referred to as z projects). The question then becomes: Under what conditions is the value of the operations equal to the sum of these two components? It is clear from the discussion in Section 4 that those conditions are restrictive. To begin with, the (parsimonious) AEG model is applicable only to steady-state situations. The Gordon growth formula is one special case, but lumpy growth projects with depreciation of PPE by the annuity method can also be accommodated. This means that the AEG model is a substantial generalization of the Gordon formula.

Sections 3 and 4 are concerned with the original AEG model as exposed in OJ 2005. The only embellishments that we have furnished are the specifications of bottom-line earnings (operating earnings minus debt interest), dividends (free cash flow minus debt interest plus debt increase), and free cash flow (k times operating earnings), together with the clean surplus principle. There is no distinction (apart from the dependence on the clean surplus relationship) between a firm-level and an equity-level model, if there is only one interest rate (i. e., $\rho_u = r$). The reason is that economic value added relative to the operating assets from the z projects (in the firm-level model) is then equal to abnormal earnings (residual income) relative to the equity from the same projects (in the equity-level model).

The AEG model was extended in Section 5 to incorporate two exogenous interest rates, the required unlevered rate of return on the equity, and the borrowing rate. The required rate of return on the equity under partial debt financing then follows from MM's Proposition 2. Again the value of the equity shows up as discounted free cash flows minus initial debt. Because of the focus on operating earnings and free cash flows, the model in Section 5 was referred to as firm-level. There is free cash flow as well as dividend policy irrelevance, since the equity value does not depend on the debt policy parameters α_1, α_2, α_3, α_4, and α_5 or the free cash flow parameter k.

Section 6 derived the related equity-level model, focused on bottom-line earnings and dividends, where the equity value is directly obtained by discounting dividends at the required rate of return on the equity under partial debt financing. The resulting formula $x_1/\bar{\rho}_e + z_1^*/(\bar{\rho}_e - g)$ looks similar to the corresponding formula $x_1/r + z_1/(r - g)$ for the original AEG model. However, dividend policy irrelevance holds only in a limited sense. The reason is that successive debt levels D_t must be set in a very particular fashion, to ensure that the required rate of return ρ_e is constant over time and equal to $\bar{\rho}_e$ as specified in equation (15). The discussion in Section 6 was centered on the equivalence between the firm-level and equity-level models, in particular the relation of z_1^* in the latter model to z_1 in the former.

The focus on predicting earnings rather than dividends (or free cash flows) is an attractive feature of the AEG model, as was mentioned in the introduction. The rationale underlying the AEG model is presumably the availability of analysts' forecasts of dividends and bottom-line earnings one and two years out. With such forecasts, one can estimate z_1^* according to the first equation in (14), as suggested in the previous section. Assuming that z_1^* increases by g from year to year, one obtains the parsimonious equity-level AEG model. It is of course a drawback that the estimated z_1^* depends on the dividend policy. Also, the meaning of z^* projects in the equity-level model is somewhat diffuse. As shown in Section 6, z_t^* is different from z_t and hence reflects not only discounted economic value added from the t-th growth project, but also an inconsistency in the valuation of zero-NPV projects.

In conclusion, we are more favorably inclined towards the firm-level model. It is more solidly based on the firm's concrete situation, including actually existing growth projects. Admittedly, it suffers from the disadvantage that analysts' forecasts of operating earnings and free cash flows are not as readily available as forecasts of bottom-line earnings and dividends. However, the firm-level AEG model is an interesting candidate for continuing value in firm valuation models of the discounted cash flow variety, since it provides a compact value formula $ox_1/\rho_u + z_1/(\rho_u - g)$ for the firm's operations and yet is a substantial extension of the Gordon growth formula. It is our final suggestion that this could be a worthwhile application for the AEG model.

Appendix

Discounting of expected dividends in Section 2

Write the equation system (3) as follows

$$\begin{pmatrix} x_{t+1} \\ z_{t+1} \\ d_{t+1} \end{pmatrix} = \begin{pmatrix} 1+r & r & -r \\ 0 & 1+g & 0 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} x_t \\ z_t \\ d_t \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_t \\ z_t \\ d_t \end{pmatrix},$$

E_1 is equal to the last (third) element of the matrix by column vector multiplication[14] ($\mathbf{I}$ is the identity matrix):

$$\frac{1}{1+r}\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\mathbf{H}\right)^t\right]\begin{pmatrix} x_1 \\ z_1 \\ d_1 \end{pmatrix} = \frac{1}{1+r}\left(\mathbf{I} - \frac{1}{1+r}\mathbf{H}\right)^{-1}\begin{pmatrix} x_1 \\ z_1 \\ d_1 \end{pmatrix}$$

$$= \left((1+r)\mathbf{I} - \mathbf{H}\right)^{-1}\begin{pmatrix} x_1 \\ z_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} \frac{1+r-c_3}{c_1 r} & \frac{1+r-c_2-c_3}{c_1(r-g)} & \frac{-1}{c_1} \\ 0 & \frac{1}{r-g} & 0 \\ \frac{1}{r} & \frac{1}{r-g} & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ z_1 \\ d_1 \end{pmatrix}. \quad (23)$$

[14]Cf. Goldberg 1958 (p. 237) or Sydsæter et al. 2005 (p. 414) on the first equality in (23). Cf. also Ohlson 1995 (p. 682) and Ohlson, Ostaszewski and Gao 2005 for applications of this discounting operation.

The result is

$$E_1 = \frac{x_1}{r} + \frac{z_1}{r-g}.$$

Discounting of expected dividends in Section 3

Writing (7) in matrix form:

$$\begin{pmatrix} ox_{t+1} \\ z_{t+1} \\ f_{t+1} \\ D_{t+1} \\ d_{t+1} \end{pmatrix} = \mathbf{H} \begin{pmatrix} ox_t \\ z_t \\ f_t \\ D_t \\ d_t \end{pmatrix},$$

where

$$\mathbf{H} = \begin{pmatrix} 1+r & r & -r & 0 & 0 \\ 0 & 1+g & 0 & 0 & 0 \\ k(1+r) & kr & -kr & 0 & 0 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \end{pmatrix},$$

and β_1, β_2, β_3, β_4, and β_5 as given by the right hand side of (8). The value of the discounted expected dividends is obtained as the last (fifth) element of the matrix times column vector multiplication

$$\frac{1}{1+r}\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\mathbf{H}\right)^t\right] \begin{pmatrix} ox_1 \\ z_1 \\ f_1 \\ D_1 \\ d_1 \end{pmatrix} = ((1+r)\mathbf{I} - \mathbf{H})^{-1} \begin{pmatrix} ox_1 \\ z_1 \\ f_1 \\ D_1 \\ d_1 \end{pmatrix},$$

where

$$((1+r)\mathbf{I} - \mathbf{H})^{-1} =$$

$$\begin{pmatrix} \frac{1+r(1+k)}{rk(1+r)} & \frac{1}{k(r-g)} & \frac{-1}{k(1+r)} & 0 & 0 \\ 0 & \frac{1}{r-g} & 0 & 0 & 0 \\ \frac{1}{r} & \frac{1}{r-g} & 0 & 0 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 \\ \left(\frac{1}{r} - \frac{\alpha_1}{1+r}\right) & \left(\frac{1}{r-g} - \frac{\alpha_2}{1+r}\right) & -\left(\frac{1}{1+r} + \frac{\alpha_3}{1+r}\right) & -\frac{\alpha_4}{1+r} & \left(\frac{1}{1+r} - \frac{\alpha_5}{1+r}\right) \end{pmatrix},$$

with

$$\gamma_1 = \frac{\alpha_1(1+r(1+k)) + \alpha_3(1+r)k + \alpha_5(1+r)k - \alpha_1\alpha_5 rk}{(1+r-\alpha_4)r(1+r)k},$$

$$\gamma_2 = \frac{\alpha_1(1+r) + (\alpha_2+\alpha_3+\alpha_5)(1+r)k - \alpha_2\alpha_5(r-g)k}{(1+r-\alpha_4)(1+r)(r-g)k},$$

$$\gamma_3 = -\frac{\alpha_1 + \alpha_5(1+\alpha_3)k}{(1+r-\alpha_4)(1+r)k},$$

$$\gamma_4 = \frac{1+r-\alpha_4\alpha_5}{(1+r-\alpha_4)(1+r)},$$

$$\gamma_5 = \frac{\alpha_5(1-\alpha_5)}{(1+r-\alpha_4)(1+r)}.$$

This gives the value of the equity E_1 at the beginning of year 1:

$$E_1 = \left(\frac{1}{r} - \frac{\alpha_1}{1+r}\right) ox_1 + \left(\frac{1}{r-g} - \frac{\alpha_2}{1+r}\right) z_1 - \left(\frac{1}{1+r} + \frac{\alpha_3}{1+r}\right) f_1 - \frac{\alpha_4}{1+r} D_1$$

$$+ \left(\frac{1}{1+r} - \frac{\alpha_5}{1+r}\right) d_1$$

$$= \frac{ox_1}{r} + \frac{z_1}{r-g} + \frac{1}{1+r}(d_1 - f_1 - D_2) = \frac{ox_1}{r} + \frac{z_1}{r-g} - D_1.$$

Discounting of free cash flows in Section 3

Since ox_{t+1}, z_{t+1}, and f_{t+1} in (7) do not depend on D_t and d_t, we have:

$$\begin{pmatrix} ox_t \\ z_t \\ f_t \end{pmatrix} = \begin{pmatrix} 1+r & r & -r \\ 0 & 1+g & 0 \\ k(1+r) & kr & -kr \end{pmatrix}^{t-1} \begin{pmatrix} ox_1 \\ z_1 \\ f_1 \end{pmatrix}. \quad (24)$$

There are two cases, $g \neq r(1-k)$ and $g = r(1-k)$.

Solution to (24), case 1: $g \neq r(1-k)$

$$\begin{pmatrix} 1+r & r & -r \\ 0 & 1+g & 0 \\ k(1+r) & kr & -kr \end{pmatrix}^{t-1} = \begin{pmatrix} 1 & \frac{-r}{r(1-k)-g} & 1 \\ 0 & 1 & 0 \\ k & \frac{-kr}{r(1-k)-g} & \frac{1+r}{r} \end{pmatrix}$$

$$\times \begin{pmatrix} (1+r(1-k))^{t-1} & 0 & 0 \\ 0 & (1+g)^{t-1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1+r}{1+r(1-k)} & \frac{r}{r(1-k)-g} & \frac{-r}{1+r(1-k)} \\ 0 & 1 & 0 \\ \frac{-kr}{1+r(1-k)} & 0 & \frac{r}{1+r(1-k)} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1+r}{1+r(1-k)}(1+r(1-k))^{t-1} & \frac{r}{r(1-k)-g}(1+r(1-k))^{t-1} - \frac{r}{r(1-k)-g}(1+g)^{t-1} & \frac{-r}{1+r(1-k)}(1+r(1-k))^{t-1} \\ 0 & (1+g)^{t-1} & 0 \\ \frac{1+r}{1+r(1-k)}k(1+r(1-k))^{t-1} & \frac{r}{r(1-k)-g}k(1+r(1-k))^{t-1} - \frac{r}{r(1-k)-g}k(1+g)^{t-1} & \frac{-r}{1+r(1-k)}k(1+r(1-k))^{t-1} \end{pmatrix},$$

where the first matrix after the first equality sign is composed of eigenvectors (columns), the second matrix contains eigenvalues on the diagonal, and the third matrix is the inverse of the eigenvector matrix.[15] Taking the third component of the matrix by column vector multiplication (24) to obtain the year's free cash flow, and using $f_t = k \cdot ox_t$, one obtains

$$f_t = \left(ox_1 + z_1 \frac{r}{r(1-k)-g} \right) k(1+r(1-k))^{t-1} - z_1 \frac{r}{r(1-k)-g} k(1+g)^{t-1}. \quad (25)$$

Discounting the free cash flows according to (25) to a present value,

$$\sum_{t=1}^{\infty} \left\{ \left(ox_1 + z_1 \frac{r}{r(1-k)-g} \right) k(1+r(1-k))^{t-1} \right.$$
$$\left. -z_1 \frac{r}{r(1-k)-g} k(1+g)^{t-1} \right\} \frac{1}{(1+r)^t} = \frac{ox_1}{r} + \frac{z_1}{r-g}.$$

As a special subcase of the first case $g \neq r(1-k)$, suppose that $z_1 = -ox_1\{(r(1-k)-g)/r\}$. Then $f_t = k \cdot ox_1(1+g)^{t-1} = f_1(1+g)^{t-1}$, and the present value of free cash flows is $k \cdot ox_1/(r-g) = f_1/(r-g)$.

Solution to (24), case 2: $g = r(1-k)$

$$\begin{pmatrix} 1+r & r & -r \\ 0 & 1+r(1-k) & 0 \\ k(1+r) & kr & -kr \end{pmatrix}^{t-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{1}{r} & 0 \\ k & k & \frac{1+r}{r} \end{pmatrix}$$

$$\times \begin{pmatrix} (1+r(1-k))^{t-1} & (t-1)(1+r(1-k))^{t-2} & 0 \\ 0 & (1+r(1-k))^{t-1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1+r}{1+r(1-k)} & -r & \frac{-r}{1+r(1-k)} \\ 0 & r & 0 \\ \frac{-kr}{1+r(1-k)} & 0 & \frac{r}{1+r(1-k)} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1+r}{1+r(1-k)}(1+r(1-k))^{t-1} & r(t-1)(1+r(1-k))^{t-2} & \frac{-r}{1+r(1-k)}(1+r(1-k))^{t-1} \\ 0 & (1+r(1-k))^{t-1} & 0 \\ \frac{1+r}{1+r(1-k)}k(1+r(1-k))^{t-1} & kr(t-1)(1+r(1-k))^{t-2} & \frac{-r}{1+r(1-k)}k(1+r(1-k))^{t-1} \end{pmatrix}.$$

Again taking the third component of (24) to obtain the year's free cash flow, one obtains

$$f_t = ox_1 k(1+r(1-k))^{t-1} + z_1 kr(t-1)(1+r(1-k))^{t-2}. \quad (26)$$

Discounting the free cash flows according to (26) to a present value,

$$\sum_{t=1}^{\infty} \left\{ ox_1 k(1+r(1-k))^{t-1} + z_1 kr(t-1)(1+r(1-k))^{t-2} \right\} \frac{1}{(1+r)^t} = \frac{ox_1}{r} + \frac{z_1}{kr}$$
$$= \frac{ox_1}{r} + \frac{z_1}{r-g},$$

since $kr = r - g$.

[15] Cf. Goldberg 1958 (for instance) on the solution to a system of homogeneous linear difference equations with constant coefficients.

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Table 1. Gordon growth formula example

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Year	Sales minus cash operating costs	Capital expendi-tures	Gross PPE	Depre-ciation	Accu-mulated depre-ciation	Net PPE	Investment in working capital	Working capital	Operating earnings ox_t	Free cash flow f_t	Initial operating earnings ox_1	Operating net assets for original operations O_1	Cumulated growth proj operating earnings ox_t-ox_1	Operating net assets for growth projects O_t-O_1
-10	30.0000	10.0000					0.2416	8.0000						
-9	30.9060	10.3020					0.2489	8.2416						
-8	31.8394	10.6131					0.2564	8.4905						
-7	32.8009	10.9336					0.2642	8.7469						
-6	33.7915	11.2638					0.2721	9.0111						
-5	34.8120	11.6040					0.2804	9.2832						
-4	35.8633	11.9544					0.2888	9.5636						
-3	36.9464	12.3155					0.2975	9.8524						
-2	38.0622	12.6874					0.3065	10.1499						
-1	39.2117	13.0706					0.3158	10.4564						
0	40.3958	13.4653					0.3253	10.7722						
1	41.6158	13.8719	118.2097	11.8210	50.2971	67.9126	0.3351	11.0975	29.7948	27.4087	29.7948	79.0102	0.0000	
2	42.8726	14.2909	121.7797	12.1780	51.8161	69.9636	0.3453	11.4327	30.6946	28.2365	29.7948	79.0102	0.8998	2.3861
3	44.1674	14.7225	125.4574	12.5457	53.3809	72.0765	0.3557	11.7780	31.6216	29.0892	29.7948	79.0102	1.8268	4.8443
4	45.5012	15.1671	129.2462	12.9246	54.9930	74.2532	0.3664	12.1337	32.5766	29.9677	29.7948	79.0102	2.7818	7.3767
5	46.8753	15.6251	133.1494	13.3149	56.6538	76.4956	0.3775	12.5001	33.5604	30.8727	29.7948	79.0102	3.7656	9.9856
6	48.2910	16.0970	137.1706	13.7171	58.3647	78.8058	0.3889	12.8776	34.5739	31.8051	29.7948	79.0102	4.7791	12.6732

Table 2. Example of lumpy project

Year	[1] Capital expenditures	[2] Investment in working capital	[3] Depreciation	[4] Accumulated depreciation	[5] Net PPE	[6] Working capital	[7] Sales minus cash operating costs	[8] Operating earnings	[9] Economic value added
1	10.0000	6.0000							
2			1.8097	0.0000	10.0000	6.0000	2.6500	0.8403	0.0403
3			1.9002	1.8097	8.1903	6.0000	2.6500	0.7498	0.0403
4			1.9952	3.7100	6.2900	6.0000	2.6500	0.6548	0.0403
5			2.0950	5.7052	4.2948	6.0000	2.6500	0.5550	0.0403
6	10.0000		2.1998	7.8002	2.1998	6.0000	2.6500	0.4502	0.0403
7			1.8097	0.0000	10.0000	6.0000	2.6500	0.8403	0.0403
8			1.9002	1.8097	8.1903	6.0000	2.6500	0.7498	0.0403
9			1.9952	3.7100	6.2900	6.0000	2.6500	0.6548	0.0403
10			2.0950	5.7052	4.2948	6.0000	2.6500	0.5550	0.0403
11	10.0000		2.1998	7.8002	2.1998	6.0000	2.6500	0.4502	0.0403