Taking Private Equity Fees Apart

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SSE/EFI Working Paper Series in Business Administration No. 2010:4

September, 2011

Abstract

In this paper I build a simplified model of the remuneration structure of a private equity fund using option theory. This model is then used to analyze the relative importance of the fixed and variable fee parts. The model is complemented with a structural model to allow for leverage in the fund portfolio companies. This increases the value of the variable fee and thus the control over the financial structure is valuable to the fund manager, hence creating a conflicting interest with the investor of the fund.

From the fund managers point of view the increased expected fee resulting from the independent control over the financial structure of the fund is at some point off-set by the increasing risk in the fund and the ability to raise borrowed capital. This creates a static trade-off point for a fund manager with one fund. For a fund manager with future fund projects, there is also a dynamic trade-off between current chosen risk, the expected fee and the ability to raise future funds.

Keywords: Investment Decisions, Private Equity, Buy-out Fund, Fund manager, Fee structure;
JEL codes: G11, G24.

Acknowledgement: I want to thank Bankforskningsinstitutet for financial support and my colleagues at the Center for Financial Analysis and Managerial Economics in Accounting and Göran Styrman for many useful comments.

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1 Introduction

The empirical evidence on the costs of investing in private equity funds[1] indicate that it is costly. The present value of all the fees charged by the mean buy-out fund studied by Metrick and Yasuda (2010) is 17.80 USD per invested 100 USD. Phalippou and Gottschalg (2009) finds that the total fee bill is more than 25 percent of the value invested, which they translate to 6 percent per year. Both these studies use actual data to evaluate the cost of private equity investments. As a comparison on the costs of investing, Khorana, Servaes and Tufano (2009) finds the total expense ratio for an average equity mutual fund in the United States to be 1.11 percent per year. Part of the fee structure for a private equity fund is typically dependent on performance and it is thus possible to use options theory to determine an expected cost of investing. I design a model based on options theory that can validate the empirical fee findings and disentangle the fee structure, i.e. separate the fixed and variable parts. Further, I combine the model with structural model to evaluate the impact of increasing leverage on the expected fee. There are number of uses for a model to calculate the expected fee of a private equity investment;

- the model can be used to get an estimate of the expected fee costs from a private equity investment,
- reversely, the model can be used by a private equity fund manager to design his fee structure,
- the model makes it possible to compare fund costs against the costs for other investments, even if the fee structure is not the same, and
- it can be used to study how the incentive structure of private equity fund managers might influence decisions on financial structure.

To the best of my knowledge there are no explicit models available to calculate the expected cost before investing in a private equity fund. There is empirical evidence and simulations for funds raised from primarily one unnamed investor between 1993 and 2006 in Metrick and Yasuda (2010). For an investor the specific fund characteristics has to be covered in the empirical data for it to be useful in an investment situation. Without a model to estimate the costs, the ex ante analysis before investing is difficult for investors and specialist fund evaluation firms (gatekeepers). It is not even certain that the empirical results of earlier studies can give guidance. The private equity industry is fairly new in its current design, so the time period studied in the empirical studies might not be representative for other investments.[2] This design is yet untested in some economic

[1]Private equity funds are venture capital or buy-out focused. Venture capital is typically investing in early stage development of firms. Buy-out funds focus on buying companies from exchanges, group companies from large groups etc.

[2]The fee structure is a way to divide gains between capital and labor. The division of capital and labor income in ways similar to the fee structure of private equity funds can be traced back in history to, at least, the Italian commenda in medieval times according to Brouwer (2005).

climates, and it is yet to be seen if the private equity model survives the current financial instability or have to be remodeled as was the case for the LBO industry in the early 90ies, which preceded the buy-out part of the private equity industry. Axelson, Strömberg and Weisbach (2009) study the financial structure of private equity funds and find that the standard financial structure of private equity investment funds maximizes the value of the fund. Their result would thus suggest that the private equity fund structure is robust.

The lack of models for quantifying the investment costs for private equity investments is a problem also for the fund manager when designing the fee structures. Different funds can have different optimal designs for their incentive structures. From an investors point of view a fund with long term safe investments could have a low fixed fee or an aggressive variable fee. The choice of fee structure gives incentives to the fund manager and in turn influences the choice of investments in the fund.

A model for calculating fees of funds with both fixed and variable fees allows for comparison between funds with different fee structures, including funds with fixed fees only. Any combination of fixed and variable fees can be transformed to fixed fee equivalents, as will be discussed (and exemplified) later on. Certainly the comparisons cannot be exact, since there are quite a few inputs that are difficult to estimate, such as the asset value process, the volatility of the fund's assets etc. However, a model that structures the complex issue should take the analysis part of the way.

The fee structure creates incentives for the private equity fund manager to make decisions on investments and financial structure. A model on fee structure makes it possible to evaluate how different choice on risk influences the income for the fund manager and reversely the cost for the investor. It is conceivable that the option like characteristics of the fee structure induces the fund manager to take on more risk either in the investments or the financial structure. The effects of increasing the financial risk are analyzed and discussed in section 3.

There are alternative ways to evaluate the costs of investing in private equity funds. One way is to do it from empirical data, as Metrick and Yasuda (2010) does, or the net effect of income and costs can be evaluated. Cumming, Siegel and Wright (2007) presents results on performance in private equity investment in their literature overview on private equity. The evidence is a bit mixed on the performance of private equity funds, for instance while Groh and Gottschalg (2006) finds positive and statistically significant alphas Phalippou and Gottschalg (2009) finds a risk adjusted under performance of 6 percent per year.[3] In contrast Gottschalg, Talmor and Vasvari (2010) finds a positive PE

[3]Phalippou and Gottschalg (2009) assumes a fixed beta of 1 when they translate returns into alphas. The financial structure of the private equity funds make this operation difficult, since the funds consists of invested capital and capital not yet invested (callable capital). Assuming that the investor of the fund has the callable capital in liquid low risk assets both the alpha and the beta are close to zero for the callable capital. This would decrease the over-all alpha and beta for the private equity investment. The investors are in fact forced to hold low risk asset, against the potential capital call, and a leveraged equity investment.

alpha. This means that the PE funds in their sample has a higher performance than a leveraged market investment.

A major problem that needs to be solved for studying private equity fund performance is that there is little data to do it from. Groh and Gottschalg (2006) creates a leveraged index from companies with similar operating performance. One conclusion the authors make is that it is important to do a comprehensive risk-adjustment that considers both operating and leverage risk for performance assessment. The adjustment for leverage does not only impact the performance of the private equity fund, but as I will show also the expected fee.

In this paper I develop a model for calculating the expected cost of investing in a private equity fund based on options theory. It is prudent to note that even if the costs might be high, it can still be an attractive investment since there is also an expected income from a private equity investment. Further, even if the cost is deducted from the fund value the cost of investment is not necessarily negatively correlated with the returns. Kaplan and Shoar (2005) find the returns of private equity investments to be persistent. I.e. a successful private equity manager tends to continue to be successful and this could motivate a higher fee for the successful manager than for a less successful manager.

The model I develop produce fees that, under realistic circumstances, produces results similar to the empirical findings of Metrick and Yasuda (2010) and Phalippou and Gottschalg (2009). The model specifically explains the result of Phalippou and Gottschalg (2009) that the fixed part of the fees is the more important one, at least in a setting where leverages of portfolio companies are close to the leverages of listed companies. The model provides a parametrization of what earlier studies have hinted at, namely that the increase of financial risk benefits the fund manager by an increase in the expected fee. The private equity fund manager can in essence set the expected fee after investors have committed their capital through fund managers' independent control over leverage in the portfolio companies. This control gives incentives for risk shifting by the private equity fund manager at the expense of the investors. Thus, there is a trade-off between the expected fee and the financial risk taken by the fund for a 'one-fund' fund manager. The model adds expected fixed and variable fees, making it possible to get fixed-fee equivalents to a fee structure that has both fixed and variable fees. This allows for fee evaluations across funds with different fee structures. The model does not allow for extensive diversification, but the effects of diversification in the funds is discussed and there might be reasons not to maximize fees in each fund. For a fund manager with future fund projects, it might not be an optimal strategy to maximize the expected fee for the current fund. The reason for this is that maximizing the expected fee can jeopardize future fund projects, hence there is a dynamic strategy in fee structures and financial choices (a dynamic trade-off between expected fees and future project incomes).

In section 2 I describe the assumptions behind the model and design the model using a fixed and one variable part. The section also contains an example, that show how costly

investing in a private equity fund is. In the next section the funds investments are levered, to mimic the use of leverage by the fund managers. The has impact on the expected cost, as described in Example 2. In section 4 the effects of portfolio diversification is described and analyzed. After this, the conclusions from the modeling are given in Section 5.

2 Model objectives and design

There are a number of choices involved in designing a model for calculating the expected fee. Each choice comes with advantages and disadvantages. I keep the model as basic a possible, while I still keep the specific characteristics of private equity funds, this to achieve the goal of estimating the expected fee cost for a private equity investment. An example of a choice is that the model is based on a call option on the fund rather than being designed as a structural model. The use of a structural model would reduce the tractability of the model in the next step, since it would require a compounded option to adjust for changes in leverage. Some potentially important aspects of the fee structure is also left out, such as catch-up provisions and clawbacks.[4] Taxes are not modeled, since different funds have different taxation schemes, and different investors abide by different taxation regulations.

2.1 The private equity fund setup

The private equity fund is not only a troublesome conglomerate, but have a different set of agreements governing its business than a normal conglomerate. Phalippou (2007) and Kaplan and Strömberg (2008) describes the general setup of a private equity firm in detail.[5] For the purpose of this paper, it is sufficient to note a few things. There are typically three parties involved in the financing of a private equity operation; the manager of the fund (the general partner or fund manager), the investors of the fund (the limited partners) and the lenders. The investments are carried through a limited partnership. At the conception of the fund the investors commit to providing capital to the fund. The fund manager then calls the committed capital on a deal by deal basis. This means that the invested capital is less than the committed capital until the fund is fully invested. The fund manager charges the fund a fixed fee (a percentage of the committed or invested capital) and a variable fee (a percentage of the profit above a hurdle rate).[6] The funds

[4] A catch-up clause means that the investor must receive his initial capital back before any variable contributions are distributed to the private equity fund manager. A clawback clause means that previously given variable fees are taken back if later deals are not sufficiently successful.

[5] Less detailed descriptions can also be found on the internet or in modern finance textbooks, such as Ross, Westerfield, Jaffe and Jordan (2008) pp570-573.

[6] The are different ways to calculate the variables fee. Some funds, for instance, charge the variable fee on realized income while others charge on estimated appreciation.

are closed end funds and are to be liquidated after a decade, but there is some flexibility as to the exact date of the funds' closing.

2.2 Model for calculating the expected fee

Metrick and Yasuda (2010) provide an in depth empirical investigation of fee types and levels for private equity funds. There are a number of fees that are charged for private equity investments, such as fixed fee (management fee), variable fee (incentive fee), transaction fee, monitoring fee and a fee covering the establishing costs. In this model, I will only include the two largest fees; the fixed and the variable fees. Thus, the total fee, as calculated here, consists of two parts; a fixed fee (fee level f_f) on net asset value (NAV) and a variable fee (fee level f_v) on the income of the fund. Note that the variable fee is dependent on the fixed fee level, since the fixed fee is deducted from the fund value. The fixed fee does not depend on anything but the current fund value.

The expected total fee is thus the sum of these two parts;

$$E\left[Fee\right] = E\left[Fixed\right] + E\left[Variable\right]. \tag{1}$$

To make it possible to calculate the expected value of the total fee some further assumptions have to be made. First, assume the only fees are the fixed and variable fees. The other fees might have some economic importance but are small in relation to the fixed and variable fees. Hunter and Jagtiani (2003), for instance, estimate an average transaction fee for an acquirer to be 0.378 percent of transaction value during 1995-2000. Second, assume the fund is fully invested at all times. If the fund is not fully invested, the estimated fee will be higher in relation to the invested capital than the model predicts. In reality private equity funds charge the fees on committed capital during an investment period (5 years is acommon time frame according to Metrick and Yasuda (2010)) and then switch to invested capital for the remainder of the funds life. The change in charging basis will typically cause the model to overestimate the charged fee since invested capital typically is less than committed capital after the investment period. There are two reason for the invested capital to be less than the committed. The fund manager does not always fill the fund with investments, and when investments are exited the capital is returned to the limited partners. Third, assume that there is no bias in the estimation of the NAV. There is no traded market for the assets of the funds, so NAVs are calculated each quarter and often not reported until a quarter later. The fact that the NAVs are used for calculating the fixed fee might give rise to suspicion as to the quality and potential bias in the NAV figures. There is a temptation not to decrease the NAV, even if warranted by general price changes, and thereby the fixed fee. Phalippou (2007) describes this problem of sluggish intermediate valuations. Any 'intentional bias' in the calculation of the NAV would tend to make the model underestimate the expected fees. Fourth, assume that the invested capital (S_t) follows a Geometric Brownian Motion (GBM);

$$dS_t = (\mu - f_f)S_t dt + \sigma S_t dW_t, \tag{2}$$

where the drift is assumed to be the risk adjusted return (μ) minus the yearly continuous fixed fee (f_f). The fixed fee is charged continuously as a simplification in the model. σ is the volatility of returns and dW is the Wiener process of the invested capital. Stock processes are often modeled using GBMs and since the assets of the private equity funds are stocks, the private equity fund can be modeled in line with this, as a portfolio of stocks. The portfolio aspects of the expected fee from investing in a private equity fund are discussed in section 4.

Under these assumptions the expected value of both the fixed and variable fees can be calculated. The expected value of the fixed fee is (trivially) contingent on the invested capital. The total fixed fee is calculated as the present value;

$$E\left[Fixed\right] = S_t\left(1 - e^{-f_f(T-t)}\right) \tag{3}$$

where proposition 4.2 from Björk (1998) has been used to calculate the expected value of the GBM for the fee. The difference between estimating the total fee as fee level times initial capital and estimating the fee using continuous compounding, as in equation (3), is minute and the simpler method should suffice in most settings. Some details on how to calculate the fixed fee are presented in Appendix A.

If we assume that the variable fee is due at the end of the funds life, the variable fee can be seen as a call option on the invested capital. Hence, there is a closed form solution at hand for;

$$E\left[Variable\right] = E\left[f_v \max\left[S_T - S_t e^{h(T-t)}, 0\right]\right] e^{-r^*(T-t)}. \tag{4}$$

where h is the hurdle rate and r^* is an unknown discount rate.[7] It is unclear what the discount rate should be, since it is does not reasonably exist a market to price this specific pay-off at T. However, for analytical purposes r^* can for instance be assumed to be zero, so the estimated costs equal the sum of all costs without discounting. If r^* is set to the risk free rate, the resulting value can be seen as the upper bound of the value of the fee, since the fee is not risk free. Noting that apart from the variable fee and the time contingent hurdle rate we have an expression close to the Black and Scholes (1973) option formula with continuous dividends (the distribution of the fixed fee f_f) which immediately gives the solution;

$$E\left[Variable\right] = f_v S_t\left(e^{(r-r^*-f_f)(T-t)} N\left(d_1\right) - e^{-r^*(T-t)} e^{h(T-t)} N\left(d_2\right)\right) \tag{5}$$

with

$$d_1 = \frac{\ln \frac{1}{e^{h(T-t)}} + (r - f_f + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}, d_2 = d_1 - \sigma\sqrt{T-t}. \tag{6}$$

Some simplified details of the calculation are presented in Appendix B. The market is incomplete in the sense that the fund is not a continuously traded security, so the analogy

[7] It is unknown here, but in a risk free world, the unknown discount rate would be the risk free rate.

with a call option is not exact. Fund shares are normally not traded other than occasionally over the counter. However, since the funds typically invest in companies that could equally well be traded on an exchange the returns and risk for a private equity fund can be replicated to some extent.[8]

Summing equation (3) and equation (5) gives an expression for the total expected fee;

$$E\left[Fee\right] = S_t\left(1 - e^{-f_f(T-t)}\right) + f_v S_t\left(e^{(r-r^*-f_f)(T-t)}N\left(d_1\right) - e^{(h-r^*)(T-t)}N\left(d_2\right)\right), \qquad (7)$$

where all inputs are defined as before. From equation (7) it is clear that the expected fee level behaves as could be anticipated. The fee level is decreasing in the hurdle rate, but increasing in invested capital, return, fixed and variable fees. Perhaps not immediately apparent, the fee level is increasing in the volatility, meaning that higher financial or operational risk increases the expected fee for the fund manager.[9] The reason increased risk also increases the expected fee of the fund manager is that the fee increases when the fund has been successful and increased in value, but does not decrease as much when the fund has had poor outcomes.

Example 1. The fee level for a private equity fund can be estimated using equation (7). For a fund with the following parameters, 2 percent fixed fee and 20 percent variable fee above a hurdle rate of eight percent, interest rate of 5 percent, an 'unknown discount rate' (r^*) set equal to the risk free rate, fund life is 10 years, and volatility of assets 25 percent, the total expected fee is 19.4 percent of invested capital.[10],[11] The asset volatility ha deliberately been set close to what can be seen as an index volatility, so as not to exaggerate the results. The sensitivity for choice of input variables is described in appendix D.

Since the total fee consists of two parts, the fixed and the variable, there are many combinations that generates the same expected fee. For the parameters in Example 1 the combinations between fixed and variable fee which give 19.4 percent are drawn in Figure 1 below. Fractions are presented on the axes. Using combinations of fixed and variable fee components, an iso-fee line can be drawn, as in Figure 1 for any choice of total cost. From equation (7) the fixed or variable fee can be calculated as a function of total fee, making it possible to calculate the iso fee lines. Fee combinations on iso fee lines that are further

[8] Cochrane (2005) find that a sample of very small Nasdaq stocks have similar return characteristics as investments in venture capital. However, the very small Nasdaq firms' returns do not explain the venture capital investment returns, suggesting they are similar phenomena, but not the same. Hwang, Quigley and Woodward (2005) develop a method for calculating return when price information is infrequent and finds lower returns than Cochrane using the same data set.

[9] The "vega risk" of the fee can be estimated using $\mathcal{V} = f_v S_t e^{(r-r^*-f_f)(T-t)} N'(d_1)\sqrt{T-t}$.

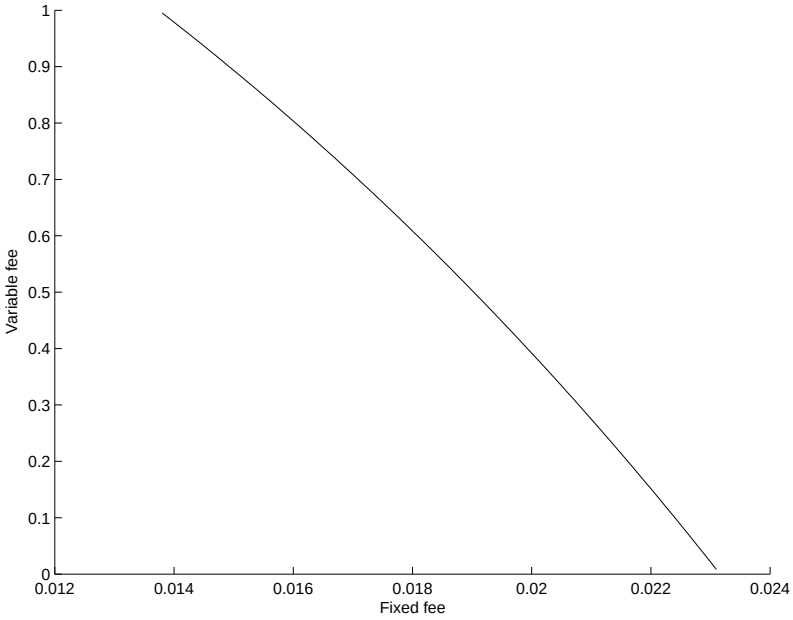
[10] Gompers and Lerner (1999*b*) finds in their sample of venture capitalists that most receive 20 percent in variable compensation. Metrick and Yasuda (2010) finds that all of their buy-out funds receive 20 percent in variable compensation. The fixed compensation varies more for the buy-out funds, where 41 percent of funds receive 2 percent and 51 percent less than 2 percent.

[11] If the unknown interest rate is set to zero, the expected fee increases to 20.3 percent.

out to the right of the drawn line are more costly for the investor. Fee combinations on iso fee lines that are below and to the left of the drawn line are less costly for the investor.

[figure 1 here]

Figure 1 Fee combinations that generate the same expected fee.



For this set of parameters the main source of income for the fund manager is the fixed fee. If the fund manager decreases the fixed fee by ten percent from 2.0 percent to 1.8 percent, the variable fee has to increase from 20.0 percent to 42.9 percent to keep the same expected fee. This relative unimportance of the variable fee corresponds well with the finding of Phalippou and Gottschalg (2009). They find that the compensation to the fund manager comes mainly from the fixed fee (management fee) and not the variable fee (incentive fee). The similarity of the empirical result and the output of the model validates the model in this respect. However, the PE fund managers can increase the financial risk of the investments, making the variable fee more valuable in expectation. This will be examined in the next section.

3 Adding leverage in the portfolio companies

A notable feature of buy-outs is that the private equity manager increases the level of debt in the financing mix of the companies they buy. According to Groh and Gottschalg (2006) leverage is then decreased during the holding period. Here the time variation of the leverage will not be considered. There are several arguments for the increase of debt

financing in the acquired companies. One argument for this increase of debt financing is that more debt increases the value of the tax shield in accordance with the reasoning of Modigliani and Miller (1958). Another argument, based on the ideas of Jensen (1986), is that a high level of debt decrease the free cash flow of the firm which keep the management of the company focused on running and optimizing the operations. However, as I will show in this analysis, there might be an additional reason. An increase in debt level also increases the expected fee for the private equity fund manager, regardless if the value of the firm increases or stays the same. The fee structure gives incentives to the fund manager to increase the risk of the investment due to the option like element of the fee structure. The result from an increase in the financial risk (leverage) is explored in this section.

Even if leverage is a notable feature, it is not the only tool that the private equity managers work with. They also work with improving the operational efficiency. Bergström, Grubb and Jonsson (2007) finds that private equity buy-out managers in Sweden between 1998 and the first half of 2006 create a significant amount of value through operational improvements. This finding indicates that the underlying asset value process might be different when an investment is acquired by a private equity fund. The model does not take this difference into account, so it might be worth noting that in calibrating the model the drift term (at least) cannot be accurately calibrated on listed companies. Given that the private equity fund manager would typically increase the operating performance this will cause my model to underestimate the fees in comparison to using the asset price development process of a listed company.

A change in leverage changes the financial risk of the investment and this is modeled using a structural model. This modeling allows for calculating the change in volatility of the investment when the leverage is changed. Assuming that the total value of the portfolio assets (V) follows a (different) GBM, the value of the equity of the PE fund is a call option on the assets of the fund and denoted $C(...)$) in line with the idea of Merton (1974).[12] Using this assumptions we can specify the relation between the (fixed) volatility of the assets (σ_v) and the volatility of the equity (σ). According to Lando (2004) p. 42 or Jennergren and Sörensson (1991);

$$\sigma = C'(V_t, \sigma_v)\sigma_v \frac{V_t}{S_t}. \tag{8}$$

When only equity is used in the financing mix, the asset volatility and the equity volatility are the same. As debt is introduced into the financing mix this equality is no longer valid and equity volatility increases in accordance with equation (8). In fact, equity volatility is strictly increasing in leverage, see appendix C for proof.

[12]Note that this is a different option than the one used to calculate the variable part of the fee. The variable part of the fee can be seen as a call option on the equity of the fund. The equity of the fund is a call option on the assets of the fund. Hence the variable part of the fee can be seen as a compound option on the assets of the fund.

Example 2. Assume that the debt to asset ratio was equal to 25 percent in Example 1. All other parameters are the same. In this example the leverage of the acquired company is increased the from 25 percent to 75 percent debt to assets.[13] To figure out what the expected fee is, we need to do this in three steps. The first step is to calculate the implicit volatility of assets from equation (8). The second step is to calculate the volatility of the invested capital after the debt to asset ratio is changed using the same equation. The third step is to apply the new volatility to equation (7) to get an estimate of the total fee after the leverage change.

From an equity volatility of 25 percent the implied asset volatility can be calculated in the first step using numerical methods to be about 18.7 percent. Using this asset volatility and changing the debt to asset ratio to 75 percent, gives a volatility for the invested capital of 56.9 percent. This new volatility substantially raises the expected variable fee from 2.3 percent to about 10.0 percent for a total of 25.7 percent in relation to the invested capital.

Increasing the leverage, as in example 2, and plotting the new trade-off line in the same way as in Figure 1 shows that we have reached a higher 'iso fee' line. The two lines have different expected fee levels (19.4 percent and 25.7 percent respectively).

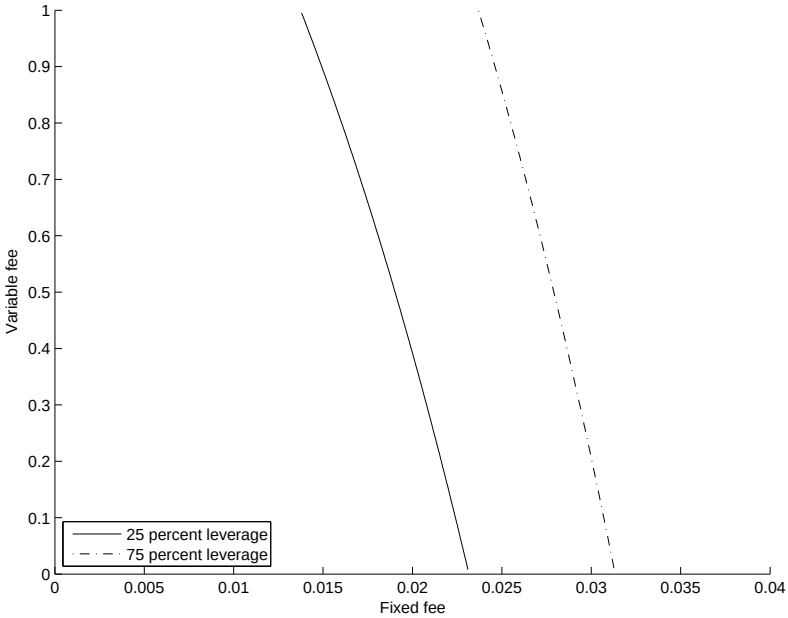
[figure 2 here]

The curve shifts to the right when the volatility is adjusted for the higher leverage, indicating a higher iso fee level. Clearly the difference in level is explained by the variable fee, since this is the part of the fee that is sensitive to changes in volatility. Perhaps not readily visible in Figure 2 is that the gap between the two iso fee lines is decreasing in the fixed fee. This is due to the fact that the risk in the second fee structure is higher. That is, substituting fixed fee for variable fee requires compensation for risk. This 'compensation' presupposes that the fund manager is risk neutral. Further, since both the fixed and the variable fee are kept at the same nominal level, it is possible to calculate what level of fixed fee corresponds to changing the leverage. Instead of raising the leverage, the private equity fund manager could raise the fixed fee to 2.8 percent per year, meaning that the shift in leverage corresponds to about 0.8 percent in expected fee per year for the fund manager.

The decisions of the private equity fund managers on financial and operational risk influences the fee they can expect to receive. From the limited partners' point of view this is a governance problem. The limited partners need to assure that the financial structures of the firms in their private equity funds are optimal for value creation in the fund and not (only) to maximize the expected fee to the fund manager. However, even if

[13] As a comparison the average market leverage for the firms in the S&P 500 industrials index was 23.5 percent at year end 2007. The median market leverage was 16.6 percent at the end of 2007. According to data from Standard & Poor's Leveraged Commentary & Data, the debt share in European buy-outs between 2003 and 2007 varied between 66.1 percent and 67.5 percent.

Figure 2 Fee combinations that generate the same expected fee for high and low leverage.



there are strong incentives for the private equity fund managers to increase the leverage in the investments, it is not necessarily always the best strategy for the fund manager to maximize leverage. There are reasons why maximum leverage is not always chosen;

- leverage puts the fee at risk for the general partner,
- lenders limit the possible leverage, and
- the general partner might want to raise additional funds in the future.

If the general partner is risk neutral then increasing the leverage is only a matter of higher expected income. If the general partner does care about putting the fee at risk, this will make it less attractive to increase the leverage. If there is a bad outcome in the investments then not only is there no variable fee, but also the fixed fee will (slowly) decrease since it is based on the invested capital. The loans needed to leverage the firms in the funds are given by banks and other lenders. The lenders set up conditions in the loan covenants to protect their interests. Dichev and Skinner (2002) presents a frequency table over different covenants for their sample of private lending agreements. The three most common covenants are 'Debt to cash Flow', 'Interest Coverage', and 'Fixed Charge Coverage'. These covenants are present in 37.7 percent, 36.7 percent and 34.4 percent of their full sample. Leverage covenants are not as common as cash flow to debt covenants. 'Debt to Tangible Net worth'and 'Leverage ratio' are present in 21.7 percent and 17.5 percent of the full sample. These figures suggest that lenders focus on the ability of the borrowing firms ability to service their debt. This is perhaps the reason why the

leverage ratios can be so high in private equity owned firms. Since the limited partner have a similar governance problem as the lenders, it could be envisioned that they would require 'covenants' that restrict the use of leverage just as the lenders. However, this does not seems to be the practice. Gompers and Lerner (1999*a*) show that fund manager performance has a drastic effect on fund raising. Both the value of equity in firms taken public the current year and previous years have a positive effect on the ability of the fund manager to raise new funds. Hence, older fund managers have a higher ability to raise new funds. This means that the reputation of a fund manager is important for success in future fund raising. This fact give the fund manager incentives not to maximize leverage, since it might jeopardize possible future income from having a good reputation. Hence, over a longer time horizon high leverage might not any longer be an optimal strategy. The first two items should give raise to a static trade-off for the financial choice of one fund. The fund manager maximizes the expected fee given the conditions set by his risk preferences and the banks willingness to lend. The third point limits the financial choices by the fund manager over time, thus creating a dynamic trade-off.

4 Portfolio diversification's effect on the expected fee

Each private equity fund invest in several companies. Until now, the analysis in this paper has been based on the invested capital as a single asset.[14] However, since the funds invest in many companies they have diversified portfolios. The diversification reduces the risk. Through this diversification the importance of the variable fee in the total fee cost of the fund investment decreases. The reason for this can be found in the "vega risk" of the fee. The vega risk is a function with a positive relation to fund return volatility (the square root of covariance). This means that diversification, which reduces the variance of the portfolio, also decreases the expected fee for the fund manager.

Diversification affects the variable part of the private equity fee calculation, and has no impact on the value of the fixed fee. The influence on the fee is carried through the variance of the portfolio returns. The calculation of the portfolio variance for the two asset case is described in finance textbooks, see for instance Ross et al. (2008) pp 286-288, and can be written;

$$\Omega_p = W\Omega W', \tag{9}$$

where W is a row vector with the portfolio weights, Ω is the covariance matrix of the portfolio assets, and Ω_p is the variance of the portfolio. The portfolio variance depends on the variance of the components and their covariances ($\Omega_{i,j}$) with each other, i.e. how the value of the different investments change together. Disregarding the highly unlikely case with perfect correlation between all portfolio assets, the diversification effect decreases the expect variability of the private equity funds portfolio, as compared to the sum of the

[14] The 'single asset' can of course be seen as the aggregated total portfolio.

variance for the individual holdings. This is of course the standard diversification effect present in any portfolio, which here decreases the fee.

Higher correlations decrease the effect of diversification, and this might be one reason for sector focus in the PE fund industry. Companies in the same sector tend to have more similar pay-off patterns then companies in different sectors. However, there are other reasons that are plausible for this focus. A sector focus benefits a PE fund manager in several ways, apart from the high correlations. A sector focus improves deal generation since not only is the fund manager familiar with the sector, but can be seen as a natural partner for other companies in the sector. The focus also helps the PE manager in the next step in the acquisition process since it improves the quality of the due diligence and enhances the ability to identify necessary strategic changes. A sector focus can also improve the chances when the PE fund manager wishes to exit the investment. It is easier to identify the right buyer if the sector is familiar and also to value the investment.

An investor can of course diversify the portfolio without the help of any specific fund manager, so the correlations in the PE fund investment might not influence the overall characteristics of the investor's portfolio. However, diversification of the portfolio outside the level of the private equity fund has no impact on the total expected fees. In the extreme case it might be preferable to invest in one PE fund with many different holdings than to invest in several PE funds. The reason for this is that there is no diversification effect on the fees outside the individual funds. The investor might, for example, be sinking well below the high watermark in one fund, while paying variable fee on another.

5 Conclusions

The model I develop in this paper describes how value is divided between on the one hand people who contribute their skills and wits and on the other hand investors who contribute their capital to joint projects. A surprising result is that the people who contribute their skills have control over the size of the remuneration they receive. The finding that the fund manager can control the expected fee level through control over the financial risk (leverage), raises the issue of potential conflicts of interest between the fund manager and the investor.

Before introducing leverage, the main part of the expected fee consists of the present value of the fixed fee. This might be a bit unexpected, since the variable fee seems high (20 percent of returns above the hurdle rate of 8 percent). This finding is in line with what Phalippou and Gottschalg (2009) finds in his empirical study. However, after adjusting for leverage, the share of the variable fee out of the total fee increases (in Example 2 it is just below 40 percent of the expected total fee). The increasing importance of the variable fee in connection with changes in leverage give the fund manager incentives to increase the financial risk. However, it might not be optimal to maximize the current expected fee since excessive risk taking might jeopardize future fund projects for the fund manager.

In essence there is a static trade-off for the fund manager in leverage. The expected enhanced profit from raising the leverage of the fund is at some point neutralized by the increase in risk.

The main reason for the fund manager not to maximize the expected return from one fund is that, given that it is profitable to manage a fund, the fund manager can jeopardize his reputation by increasing the risk of poor outcomes too much. The reputations effect is strong, since successful fund managers tend to continue to be successful, and this will put a soft limit of what the optimal long run leverage is.

A Calculating the fixed fee

Starting with the GBM of the capital;

$$dS_t = (\mu - f_f)S_t dt + \sigma S_t dW_t. \tag{10}$$

The fixed fee is calculated as the invested capital (S_t) times the fixed fee (f_f). Since the fee is fixed we can use proposition 4.2 in Björk (1998) to get the expected value of the fixed fee function at each time (index s);

$$E\left[Fixed|_{t=s}\right] = f_f S_t e^{(r-f_f)s}. \tag{11}$$

Discounting with $e^{-r(s-t)}$ to t and summing the expected fixed fee over the holding period $(T-t)$ gives us the sum of the expected fee;

$$E\left[Fixed\right] = f_f S_t \int_t^T e^{(r-f_f)(s-t)} e^{(-r)(s-t)} ds = \frac{S_t f_f}{-f_f}\left(e^{(-f_f)(T-t)} - 1\right) = S_t\left(1 - e^{(-f_f)(T-t)}\right) \tag{12}$$

B Calculating the variable fee

The variable fee is as defined in Equation (4);

$$E\left[Variable\right] = E\left[f_v \max\left[S_T - S_t e^{h(T-t)}, 0\right]\right] e^{-r^*(T-t)}. \tag{13}$$

Noting that the P-dynamics in Equation (2) can be rewritten and solved as;

$$\tilde{S}_T = S_t e^{(r-f_f-\frac{1}{2}\sigma^2)(T-t)+\sigma\sqrt{T-t}Z} \tag{14}$$

, where Z is a stochastic variable with a standard normal distribution. The closed form for the expected value of the variable fee can be calculated;

$$\begin{aligned} E[Variable] &= E\left[f_v \max\left[\tilde{S}_T - S_t e^{h(T-t)}, 0\right]\right] e^{-r^*(T-t)} = \\ &= f_v(E\left[\tilde{S}_T | \tilde{S}_T \geq S_t e^{h(T-t)}\right] - \\ &\quad -S_t e^{h(T-t)} E\left[1 | \tilde{S}_T \geq S_t e^{h(T-t)}\right]) e^{-r^*(T-t)} = \\ &= f_v\left(S_t e^{(r-f_f)(T-t)} N(d_1) - S_t e^{h(T-t)} N(d_2)\right) e^{-r^*(T-t)} = \\ &= f_v S_t \left(e^{(r-r^*-f_f)(T-t)} N(d_1) - e^{(h-r^*)(T-t)} N(d_2)\right), \end{aligned} \tag{15}$$

where

$$d_1 = \frac{\ln \frac{1}{e^{h(T-t)}} + ((r - f_f) + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}. \tag{16}$$

C Proof of increasing equity volatility in leverage

Assume alpha ($\alpha \in (0,1)$) is the equity share of the assets (V), where the assets are neither entirely financed by equity nor entirely financed by debt. Then the equity volatility is as in equation (8);

$$\sigma = N(d_1^c)\sigma_v \frac{1}{\alpha}, \tag{17}$$

where I assume that value of the equity of the fund can be seen as a standard call option. Due to this assumption, the derivative of the call option (the delta) is $N(d_1^c)$. For a standard call option where the stock price is substituted with assets and the strike price is substituted with debt; $d_1^c = \frac{\ln \frac{1}{1-\alpha} + b}{c}$. Where $b = (r + \sigma_v^2/2)(T-t)$ and $c = \sigma_v\sqrt{T-t}$. b is not part of the derivative of $N(d_1^c)$ against α and c is always positive, so neither b nor c are important for the proof. If equity volatility (σ) is decreasing in α, i.e. if

$$\frac{\partial \sigma}{\partial \alpha} < 0, \tag{18}$$

is true then equity volatility is increasing in increasing leverage. The derivative of equity volatility in leverage is,

$$\begin{aligned} \frac{\partial \sigma}{\partial \alpha} &= \sigma_v \left(\frac{\partial N(d_1^c)}{\partial d_1^c}\frac{\partial d_1^c}{\partial \alpha}\frac{1}{\alpha} - N(d_1^c)\frac{1}{\alpha^2}\right) = \\ &= \frac{\sigma_v}{\alpha}\left(\frac{\partial N(d_1^c)}{\partial d_1^c}\frac{\partial d_1^c}{\partial \alpha} - N(d_1^c)\frac{1}{\alpha}\right) = \\ &= \frac{\sigma_v}{\alpha}\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(d_1^c)^2}{2}} \frac{1}{c(1-\alpha)} - \frac{N(d_1^c)}{\alpha}\right) = \\ &= \frac{\sigma_v}{\alpha^2 c(1-\alpha)\sqrt{2\pi}}\left[\alpha e^{-\frac{(d_1^c)^2}{2}} - N(d_1^c)c(1-\alpha)\sqrt{2\pi}\right]. \end{aligned} \tag{19}$$

The first term is always positive for the allowed values of the parameters, so it can be ignored in the further analysis. Using the definition of the normal distribution gives;

$$\Rightarrow \qquad \alpha e^{-\frac{(d_1^c)^2}{2}} - \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^{d_1^c} e^{-\frac{u^2}{2}} du \right) c(1-\alpha)\sqrt{2\pi} =$$

$$= \quad e^{-\frac{(d_1^c)^2}{2}} \left(\alpha - \left(\int_{-\infty}^{d_1^c} e^{\frac{(d_1^c)^2 - u^2}{2}} du \right) c(1-\alpha) \right), \tag{20}$$

where again the first term is always positive. So if

$$\left(\int_{-\infty}^{d_1^c} e^{\frac{(d_1^c)^2 - u^2}{2}} du \right) c(1-\alpha) > \alpha$$

$$\Leftrightarrow \qquad \int_{-\infty}^{d_1^c} e^{\frac{(d_1^c)^2 - u^2}{2}} du > \frac{\alpha}{c(1-\alpha)} \tag{21}$$

holds, then the equity volatility is decreasing in increasing equity share and thus it is increasing in increasing leverage. However, this is true since the left hand term has a positive derivative[15],

$$\frac{\partial \int_{-\infty}^{d_1^c} e^{\frac{(d_1^c)^2 - u^2}{2}} du}{\partial \alpha} > 0 \tag{22}$$

the right hand side has a negative derivative,

$$\frac{\partial \frac{\alpha}{c(1-\alpha)}}{\partial \alpha} = \frac{1}{c} \left(\frac{1}{1-\alpha} - \frac{1}{(1-\alpha)^2} \right) < 0 \tag{23}$$

, and for $\alpha \to 0$ expression (21) is fulfilled at the limit. So equity volatility is strictly monotonically decreasing in equity to assets and thus equity volatility is strictly monotonically *increasing* in debt to assets.

D Input variable sensitivity

The input variables selected for the example are reasonably in line with what is common for PE funds, but they are by no means fixed. The impact on the expected fee from changing one variable at a time can, for Example 1, be seen in Table 1. In this table I first show both how much the expected fee changes when one variable is changed by -10, -5, 0, +5, and +10 percent and then what the variables needs to be to produce an expected fee that is 10 percent higher or lower. The idea with the first set of columns is to show the sensitivity of the expected fee against the input variables. I calculate the variable needed in the last two columns to illustrate the relative importance of each variable. I.e. The effect of changing the fixed fee from 2 percent to 1.75 percent is the same as changing the volatility from 25 percent to 2.4 percent.

[15] u^2 is smaller than or equal to $(d_1^c)^2$ and thus the expression is always positive.

Table 1. Sensitivity analysis for Example 1

Variable	Fee level with change in variable					Variable value for fee change	
	-0.10	-0.05	0.00	0.05	0.10	-10 percent	+10 percent
Interest	0.2036	0.2051	0.2067	0.2085	0.2103	0.0000	0.0739
Hurdle rate	0.2102	0.2084	0.2067	0.2051	0.2036	0.1608	0.0375
Volatility	0.2018	0.2043	0.2067	0.2093	0.2118	0.1314	0.3508
Fixed fee	0.1916	0.1992	0.2067	0.2142	0.2217	0.0173	0.0228
Variable fee	0.2042	0.2055	0.2067	0.2080	0.2093	0.0377	0.3623
End time	0.1902	0.1985	0.2067	0.2149	0.2229	8.7534	11.2867

In this table the first five columns presentes the expected fee level when one input is changed. The last two columns decribe the level of the parameter that results in a +/- 10 percent change in fee level.

The relative importance of the fixed fee can be seen also in Table 1, both direct in the fixed fee row and indirect in the end time row. In Table 2, the same sensitivity analysis is presented for Example 2.

Table 2. Sensitivity analysis for Example 2

Variable	Fee level with change in variable					Variable value for fee change	
	-0.10	-0.05	0.00	0.05	0.10	-10 percent	+10 percent
Interest	0.2632	0.2661	0.2690	0.2721	0.2752	0.0237	0.0698
Hurdle rate	0.2716	0.2703	0.2690	0.2677	0.2664	0.1634	-0.0000
Volatility	0.2590	0.2641	0.2690	0.2738	0.2784	0.4235	0.7487
Fixed fee	0.2549	0.2620	0.2690	0.2760	0.2829	0.0162	0.0239
Variable fee	0.2603	0.2646	0.2690	0.2734	0.2778	0.1387	0.2613
End time	0.2508	0.2600	0.2690	0.2779	0.2866	8.5358	11.5508

In this table the first five columns presentes the expected fee level when one input is changed. The last two columns decribe the level of the parameter that results in a +/- 10 percent change in fee level.

References

Axelson, U., Strömberg, P. and Weisbach, M. S. (2009), 'Why are buyouts levered? The financial structure of private equity funds', *The Journal of finance* **64**(4), 1549–1582.

Bergström, C., Grubb, M. and Jonsson, S. (2007), 'The operating impact of buyouts in Sweden: A study of value creation', *The Journal of Private Equity* **11**(1), 22–39.

Björk, T. (1998), *Arbitrage Theory in Continuous Time*, Oxford University Press, Great Clarendon Street, Oxford OX2 6DP, Great Britain.

Black, F. and Scholes, M. (1973), 'The pricing of options and corporate liabilities', *The Journal of Political Economy* **81**(3), 637–654.

Brouwer, M. (2005), 'Managing uncertainty through profit sharing contracts from medieval Italy to Silicon Valley', *Journal of Management and Governance* **9**, 237–255.

Cochrane, J. H. (2005), 'The risk and return of venture capital', *Journal of Financial Economics* **75**, 3–52.

Cumming, D., Siegel, D. S. and Wright, M. (2007), 'Private equity, leveraged buyouts and governance', *Journal of Corporate Finance* **13**, 439–460.

Dichev, I. D. and Skinner, D. J. (2002), 'Large-sample evidence on the debt covenant hypothesis', *Journal of Accounting Research* **40**(4), 1091–1123.

Gompers, P. A. and Lerner, J. (1999*a*), 'What drives venture capital fundraising?', *SSRN eLibrary* .

Gompers, P. and Lerner, J. (1999*b*), 'An analysis of compensation in the U.S. venture capital partnership', *Journal of Financial Economics* **51**, 3–44.

Gottschalg, O., Talmor, E. and Vasvari, F. (2010), 'Private equity fund level return attribution: Evidence from U.K. based buyout funds', *London Business School* .

Groh, A. P. and Gottschalg, O. (2006), 'The risk-adjusted performance of US buyouts', *23rd International Symposium on Money, Banking and Finance, Université Charles de Gaulle - Lille 3 (France), June 2006* .

Hunter, W. C. and Jagtiani, J. (2003), 'An analysis of advisor choice, fees, and effort in mergers and acquisitions', *Review of Financial Economics* **12**, 65–81.

Hwang, M., Quigley, J. M. and Woodward, S. E. (2005), 'An index for venture capital, 1987-2003', *Contributions to Economic Analysis & Policy* **4**(1).

Jennergren, L. P. and Sörensson, T. (1991), 'Choice of model in convertible valuation - a case study', *Omega* **19**(4), 185–195.

Jensen, M. C. (1986), 'Agency costs of free cash flow, corporate finance, and takeovers', *The American Economic Review* **76**(2), 323–329.

Kaplan, S. N. and Shoar, A. (2005), 'Private equity performance: Returns, persistance, and capital flows', *The Journal of Finance* **60**(4), 1791–1823.

Kaplan, S. N. and Strömberg, P. (2008), 'Leveraged buyouts and private equity', *National Bureau of Economic Research* **14207**.

Khorana, A., Servaes, H. and Tufano, P. (2009), 'Mutual funds fees around the world', *The Review of Financial Studies* **22**(3), 1279–1310.

Lando, D. (2004), *Credit Risk Modeling*, Princeton Series in Finance, Princeton University Press, 41 William Street, Princeton, New Jersey 08540, United States of America.

Merton, R. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', *Journal of Finance* **29**, 449–470.

Metrick, A. and Yasuda, A. (2010), 'The economics of private equity funds', *The Review of Financial Studies* **23**(6), 2303–2341.

Modigliani, F. and Miller, M. H. (1958), 'The cost of capital, corporation finance and the theory of investment', *The American Economic Review* **48**(3), 261–297.

Phalippou, L. (2007), 'Investing in private equity funds: A survey', *SSRN eLibrary* .

Phalippou, L. and Gottschalg, O. (2009), 'The performance of private equity funds', *The Review of Financial Studies* **22**(4), 1747–1776.

Ross, S. A., Westerfield, R. W., Jaffe, J. and Jordan, B. D. (2008), *Moderns Financial Management*, McGraw-Hill/Irwin, 1221 Avenue of the Americas, New York, NY, 10020, Unites States of America.