The Conventional Formula for the Nominal Growth Rate of Free Cash Flows is OK – A Comment on Three Recent Papers in the *Journal of Applied Corporate Finance*

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Abstract

The conventional formula for the nominal growth rate of free cash flows (equal to dividends when there is no interest-bearing debt) says that this growth rate is equal to the product of the plowback ratio and the nominal rate of return on the assets (the latter equal to book equity when there is no debt). In a recent issue of the *Journal of Applied Corporate Finance*, M. Bradley and G. A. Jarrell claim that the conventional formula is wrong when there is positive inflation, proposing instead an alternative formula. In a rejoinder to that paper in the same journal, G. Friedl and B. Schwetzler assert that the conventional formula is right. In a comment on Friedl and Schwetzler, Bradley and Jarrell reassert their original position, that is, the conventional formula is wrong and the alternative one is right. This note shows that the conventional formula is right and that both formulas give the same nominal growth rate. Consequently, both are OK.

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Introduction

The Gordon growth model for share valuation that can be found in virtually all corporate finance texts assumes that dividends will increase at the growth rate G, starting from the dividend DIV_1 that is forecasted for the end of year 1. The value V_0 of one share of stock at the end of year 0 is then $V_0 = DIV_1 / (W – G)$, where W is the required rate of return on the equity. All components are in nominal terms. In particular, the growth rate G should also be nominal, in other words including inflation.

The calculation of G is discussed in a recent paper in the *Journal of Applied Corporate Finance* by Michael Bradley and Gregg A. Jarrell (subsequently abbreviated as B & J).[2] In particular, these authors severely criticize the commonly suggested formula that sets G equal to the plowback ratio multiplied by the return on equity. Consider the following example from the textbook by Brealey, Myers and Allen[3] that is also cited by B & J[4]: Cascade's plowback ratio is equal to (DIV obviously means dividend, and EPS earnings per share)

Plowback ratio = 1 – payout ratio = 1 – (DIV / EPS) = 0.44.

This is the usual accounting definition of plowback (or retention) ratio. Moreover, Cascade's return on equity is

Return on equity = EPS / (book equity per share) = 0.12.

[2] B & J 2008.
[3] Brealey, Myers and Allen 2006, pp. 66-67.
[4] B & J 2008, p. 69.

The book equity will then increase by $0.44 \times 0.12 = 5.3\%$. Earnings and dividends per share will also increase by 5.3%. In other words, $G = 0.44 \times 0.12 = 5.3\%$. B & J argue that this formula, referred to here as *the conventional formula*, is wrong, since the impact of expected inflation is not taken into account properly.[5] Instead, they propose an alternative formula. In a subsequent issue of the same journal, Gunther Friedl and Bernhard Schwetzler (subsequently abbreviated as F & S) argue that the conventional formula is right and that the alternative formula results in an estimate of G that is different from that provided by the conventional formula.[6] In a comment on F & S, B & J again assert that the conventional formula is wrong and the alternative formula is right.[7]

The purpose of this comment is to show that the conventional formula that says that G is equal to the product of plowback ratio and return on equity is right. The alternative formula is also right, in fact, equivalent to the first one. So both are OK.

A couple of assumptions are listed right away: The company has only one share, so the share value is equal to the value of the total equity. The company has no debt. This means that dividend is equal to free cash flow. Consequently, this note will talk about free cash flows rather than dividends. The company has no working capital, only property, plant and equipment (PPE) with a long economic life. Also, there are no taxes. These assumptions are for simplicity only, without loss of generality, and are also made by B & J. Further assumptions are mentioned below.

[5] They write: "Not only is this expression for the nominal growth rate not self-evident, it is incorrect as we demonstrate …" (B & J 2008, p. 69).
[6] F & S 2011, pp. 109-111. In reaching this conclusion about the B & J alternative formula, F & S apparently disregard the fact that B & J apply a definition of plowback ratio that is different from the usual accounting definition of that ratio (cf. below).
[7] B & J 2011.

Table 1 **Alternative calculations of G, the nominal growth rate of free cash flows**

Common assumptions

$g = 2\%$, $\Pi = 1\%$, $CASHNETRV_0 = 1.5$, $CAPX_0 = 1.0000$

First case: Linear depreciation

Year	Cohort purchase price	End of year -1 rest value	End of year 0 rest value
-5	0.8618	0.1724	0.0000
-4	0.8878	0.3551	0.1776
-3	0.9146	0.5488	0.3658
-2	0.9422	0.7538	0.5653
-1	0.9707	0.9707	0.7765
0	1.0000		1.0000

$DEPR_0 = (0.1724-0.0000) + (0.3551-0.1776) + (0.5488-0.3658) + (0.7538-0.5653) + (0.9707-0.7765)$	0.9154
$K_{-1} = 0.1724+0.3551+0.5488+0.7538+0.9707$	2.8007
$K_0 = 0.1776+0.3658+0.5653+0.7765+1.0000$	2.8853
$NNI_0 = CAPX_0 - DEPR_0 = K_0 - K_{-1}$	0.0846
$ERN_0 = CASHNETRV_0 - DEPR_0$	0.5846
$FCF_0 = ERN_0 - NNI_0$	0.5000
Plowback ratio = $[1 - (FCF_0 / ERN_0)] = [NNI_0 / ERN_0]$	14.47 %
Return on assets = $[ERN_0 / K_{-1}]$	20.87 %
Conventional formula: G = (plowback ratio) × (return on assets)	3.02 %
$r = ERN_0 / [K_{-1}(1+\Pi)]$	20.67 %
$k = [K_0 - K_{-1}(1+\Pi)] / ERN_0$	9.68 %
B & J formula: $G = (1+\Pi) \times r \times k + \Pi$	3.02 %

Second case: Economic depr.

Year	Cohort purchase price	End of year -1 rest value	End of year 0 rest value
-5	0.8618	0.2026	0.0000
-4	0.8878	0.4174	0.2087
-3	0.9146	0.6450	0.4300
-2	0.9422	0.8859	0.6644
-1	0.9707	0.9707	0.9127
0	1.0000		1.0000

$DEPR_0 = (0.2026-0.0000) + (0.4174-0.2087) + (0.6450-0.4300) + (0.8859-0.6644) + (0.9707-0.9127)$	0.9057
$K_{-1} = 0.2026+0.4174+0.6450+0.8859+0.9707$	3.1215
$K_0 = 0.2087+0.4300+0.6644+0.9127+1.0000$	3.2158
$NNI_0 = CAPX_0 - DEPR_0 = K_0 - K_{-1}$	0.0943
$ERN_0 = CASHNETRV_0 - DEPR_0$	0.5943
$FCF_0 = ERN_0 - NNI_0$	0.5000
Plowback ratio = $[1 - (FCF_0 / ERN_0)] = [NNI_0 / ERN_0]$	15.86 %
Return on assets = $[ERN_0 / K_{-1}]$	19.04 %
Conventional formula: G = (plowback ratio) × (return on assets)	3.02 %
$r = ERN_0 / [K_{-1}(1+\Pi)]$	18.85 %
$k = [K_0 - K_{-1}(1+\Pi)] / ERN_0$	10.61 %
B & J formula: $G = (1+\Pi) \times r \times k + \Pi$	3.02 %

A counterexample: The conventional formula is OK

See the first case of Table 1! The notation is quite close to that of B & J. All action takes place at year ends. We are currently at the end of year 0. Revenue minus cash costs at the end of year 0, denoted $CASHNETRV_0$, is 1.5. The (yearly) real growth rate is g = 2%, and inflation Π = 1%, so the nominal growth rate G is (1+g)(1+Π) – 1 = 3.02%. The company is assumed to be in a steady state, meaning that revenue minus cash costs, capital expenditures, PPE (gross and net),[8] and depreciation are all increasing at the nominal rate 3.02%.

The company uses PPE with an economic life of 5 years. The last PPE cohort has just been acquired at the end of year 0, at the purchase price 1. In other words, capital expenditures at the end of that year, denoted by $CAPX_0$, are 1. Nominal purchase prices of PPE cohorts, acquired at the ends of years 0, -1 … -5, are shown in the first column of the table. These purchase prices increase by the nominal growth rate 3.02%, due to both real growth and inflation. It is assumed in the first case that the PPE is depreciated linearly over the economic life of 5 years. Under that assumption, nominal rest values (i. e., after depreciation so far) for cohorts that have not been retired are shown in the second column for the end of year -1, and in the third column for the end of year 0. In particular, it is seen that the cohort that was acquired at the end of year -5 at a price of 0.8618 has just been retired at the end of year 0. Depreciation at the end of year 0, denoted by $DEPR_0$, is $(1/5)\times(0.8618+0.8878+0.9146+0.9422+0.9707) = (0.1724–0.0000) + (0.3551–0.1776) + (0.5488–0.3658) + (0.7538–0.5653) + (0.9707–0.7765) = 0.9154$. Net PPE at the end of year -1, denoted by K_{-1}, is obtained by adding up the individual rest values in the second column and is found to be 2.8007 (cf. footnote 8). Similarly, net PPE at the end of year 0, denoted by K_0, is 2.8853. Net new investment at the end of year 0, denoted by NNI_0, is equal to capital expenditures minus depreciation, i. e., $NNI_0 = CAPX_0 – DEPR_0 = K_0 – K_{-1} = 0.0846$. This equation follows immediately from the accounting definition of capital expenditures that is equal to depreciation plus increase in net PPE. In other words, $CAPX_0 = DEPR_0 + (K_0 – K_{-1})$. This definition is valid if the clean surplus condition holds (i. e., if PPE is not written up or down directly

[8] The accounting measure gross PPE is the sum of purchase prices of non-retired cohorts. In Table 1, gross PPE at the end of year -1 is equal to (0.8618 + 0.8878 + 0.9146 + 0.9422 + 0.9707) (the economic life is 5 years). Net PPE is the sum of individual cohort rest values (after depreciation so far) and is equal to (0.1724 + 0.3551 + 0.5488 + 0.7538 + 0.9707) at the end of the same year in the first case in Table 1.

against owners' equity; that assumption holds in Table 1). Accounting earnings at the end of year 0, denoted by ERN_0, are $ERN_0 = CASHNETRV_0 - DEPR_0 = 1.5 - 0.9154 = 0.5846$. Free cash flow at the end of year 0, denoted by FCF_0, is $FCF_0 = ERN_0 - NNI_0 = 0.5$. It is again mentioned that free cash flow equals dividend, since there is no debt.

Suppose now that an outside analyst (who does not know the details of the first case of Table 1) is interested in estimating the nominal growth rate G. The historical accounting information that he/she would have at hand is the payout ratio FCF_0 / ERN_0, and consequently the plowback ratio $1 - (FCF_0 / ERN_0) = NNI_0 / ERN_0$ (the usual accounting definition of plowback ratio). The outside analyst would also have at hand the return on assets (equal to the return on equity, again since there is no debt) ERN_0 / K_{-1}. With this information, G can be estimated by the conventional formula, as in the previous example from Brealey et al., i. e. the plowback ratio multiplied by the return on assets:

$$G = [1 - (FCF_0 / ERN_0)] \times (ERN_0 / K_{-1}) = (NNI_0 / ERN_0) \times (ERN_0 / K_{-1}).$$

Apparently, G must be equal to $NNI_0 / K_{-1} = (K_0 - K_{-1}) / K_{-1}$, since that is the company's true nominal growth rate that applies not only to the PPE, but also (by assumption) to revenue minus cash costs, capital expenditures, and depreciation; and hence also to earnings and free cash flows.

Hence, the conventional formula provides the right nominal growth rate of free cash flows. The conventional calculation of G is exemplified in the first case of Table 1. With the assumptions made, the nominal growth rate G is found to be 3.02%, as it should.

The conclusion that the conventional formula is OK in no way depends on the choice of depreciation method. Linear depreciation was selected merely for simplicity. The only critical assumption is that the clean surplus condition holds.

B & J emphasize that they assume *economic depreciation*, meaning the amount of depreciation that is required to maintain the same real productive capacity of the firm (B & J 2008, p. 76; B & J 2011, p. 113). The second case of Table 1 assumes economic depreciation. The economic life of the PPE is still assumed to be 5 years. At the end of year 0, the cohort that is being replaced is the one that was acquired at the end of year -5 at the purchase price 0.8618. Replacing this cohort 5 years later requires a nominal purchase outlay of $0.8618 \times (1+\Pi)^5 = 0.8618 \times (1+0.01)^5 = 0.9057$. In other words, economic depreciation at the end of year 0 amounts to 0.9057. The second case of Table 1 shows end of year -1 and end of year 0 cohort rest values under one particular depreciation pattern for individual cohorts that results in total depreciation for the entire stock of PPE being equal to economic depreciation, i. e., to the nominal capital expenditure for replacing that cohort that was acquired 5 years ago and is currently being retired.[9] Merely working out the various definitions from the first case under the assumption of economic rather than linear depreciation once again indicates that the nominal growth rate G according to the conventional formula must be 3.02%. Capital expenditures and free cash flow obviously do not depend on the depreciation, but the breakdown of capital expenditures into depreciation and increase in net PPE differs between the first and second cases in Table 1. The plowback ratio and the return on assets do change, but in a counteracting manner, so that the

[9] There are many different depreciation patterns for individual cohorts that result in total depreciation for the entire stock of PPE being equal to economic depreciation. The particular pattern in the second case of Table 1 was constructed by assuming linear depreciation of a fictitious purchase price (p × a) for all years of the economic life of a cohort, except the first year where the depreciation is [a – (4/5)(p × a)]. The true nominal purchase price of the cohort is a, and p is a scaling factor such that depreciation for the entire stock of PPE becomes equal to economic depreciation. (In the second case of Table 1, p is equal to 1.1753.)

estimated nominal growth rate G remains the same. In other words, the assumption of economic depreciation does not rule out the conventional formula for the calculation of G.

B & J criticize a number of textbook authors for not *proving* the conventional formula, apparently for the reason that these authors consider that formula as self-evident.[10] It is also the opinion of the present author that the conventional formula is fairly evident.

The B & J formula is also OK

According to B & J, the right formula for G is

$$(1+\Pi) \times r \times k + \Pi = (1+\Pi) \times (ERN_0 / [K_{-1}(1+\Pi)]) \times ([K_0 - K_{-1}(1+\Pi)] / ERN_0) + \Pi,$$

where the factors r and k are implicitly defined after the equality sign.[11] r can be interpreted as *real* return on assets. k is the plowback ratio as defined by B & J. It is noted that k is not the same as the usual accounting definition of plowback ratio. Rewriting the last term Π, the right hand side is equal to

$$(1+\Pi) \times (ERN_0 / [K_{-1}(1+\Pi)]) \times ([K_0 - K_{-1}(1+\Pi)] / ERN_0)$$

$$+ (1+\Pi) \times (ERN_0 / [K_{-1}(1+\Pi)]) \times ([\Pi \times K_{-1}] / ERN_0)$$

$$= (1+ \Pi) \times (ERN_0 / [K_{-1}(1+\Pi)]) \times ([K_0 - K_{-1} - \Pi \times K_{-1} + \Pi \times K_{-1}] / ERN_0)$$

$$= (ERN_0 / K_{-1}) \times ([K_0 - K_{-1}] / ERN_0) = (ERN_0 / K_{-1}) \times (NNI_0 / ERN_0).$$

[10] B & J 2008, p. 69.

[11] See B & J 2008, pp. 67, equation (4); 68, equation (7); and 76, equation (5).

It is clear from the last line of this equation that the formula for G according to B & J is in fact the same as the conventional formula. This is also seen in Table 1, where the B & J calculation gives exactly the same result as the conventional formula, 3.02%, in both cases. Consequently, the B & J formula also gives the right result.

Conclusion

It is not correct to claim, as do B & J, that the conventional formula for nominal growth in free cash flows is wrong. The conventional formula is, in fact, right. F & S also arrive at that conclusion, although through a somewhat different line of argument (F & S 2011, p. 111). The alternative formula that B & J suggest is also right, actually only a different way of writing the conventional formula.

References

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