

A TUTORIAL ON THE DISCOUNTED CASH FLOW MODEL FOR VALUATION OF COMPANIES

L. Peter Jennergren *

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Abstract

All steps of the discounted cash flow model are outlined. Essential steps are: calculation of free cash flow, forecasting of future accounting data (income statements and balance sheets), and discounting of free cash flow. There is particular emphasis on forecasting those balance sheet items which relate to property, plant, and equipment. There is an exemplifying valuation included (of a company called McKay), as an illustration. A number of other valuation models (discounted dividends, adjusted present value, economic value added, and abnormal earnings) are also discussed. Earlier versions of this working paper were entitled “A Tutorial on the McKinsey Model for Valuation of Companies”.

Key words: Valuation, free cash flow, discounting, accounting data

JEL classification: G31, M41, C60

*Stockholm School of Economics, Box 6501, S - 11383 Stockholm, Sweden. The author is indebted to Tomas Hjelström, Joakim Levin, Per Olsson, Kenth Skogsvik, and Ignacio Velez-Pareja for discussions and comments. Also, the author thanks the Torsten and Ragnar Söderberg Foundations and Försäkringsbolaget Pensionsgaranti for economic support. Minor changes made on December 9, 2013.

1 Introduction

This tutorial explains all the steps of the discounted cash flow model, prominently featured in a book by an author team from McKinsey & Company (Tim Koller, Marc Goedhart, and David Wessels: *Valuation: Measuring and Managing the Value of Companies*, 5th ed. 2010). The purpose is to enable the reader to set up a complete valuation model of his/her own, at least for a company with a simple structure. The discussion proceeds by means of an extended valuation example. The company that is subject to the valuation exercise is the McKay company.¹

The McKay example in this tutorial is somewhat similar to the Preston example (concerning a trucking company) in the first two editions of *Valuation: Measuring and Managing the Value of Companies* (Copeland et al. 1990, Copeland et al. 1994). However, certain simplifications have been made, for easier understanding of the model. In particular, the capital structure of McKay is composed only of equity and debt (i. e., no convertible bonds, etc.). Also, McKay has no operating leases or capitalized pension liabilities.² McKay is a single-division company and has no foreign operations (and consequently there are no translation differences). There is no goodwill and no minority interest. The purpose of the McKay example is merely to present all essential aspects of the discounted cash flow model as simply as possible. Some of the historical income statement and balance sheet data have been taken from the Preston example. However, the forecasted income statements and balance sheets are totally different from Preston's. All monetary units are unspecified in this tutorial (in the Preston example in Copeland et al. 1990, Copeland et al. 1994, they are millions of US dollars).

This tutorial is intended as a guided tour through one particular implementation of the discounted cash flow model and should therefore be viewed only as exemplifying: This is one way to set up a valuation model. Some modelling choices that have been made will be pointed out later on. However, it should be noted right away that the specification

¹Earlier versions of this tutorial were entitled “A Tutorial on the McKinsey Model for Valuation of Companies”, since they focused on the McKinsey implementation of the discounted cash flow model. However, after several revisions of the McKinsey book as well as of this tutorial, there are now some differences in emphasis and approach between the two, motivating the title change, starting from the sixth revision of this tutorial in 2006. The most important change in the ninth revision of this tutorial is that Appendices 1 and 2 have been deleted. The previous Appendix 1 contained a table of estimated values of n (PPE economic life) and K (capital intensity), based on a data base from Statistics Sweden. This data base is by now quite old. Also, the same table has been published elsewhere (Jennergren 2008, p. 1555). The previous Appendix 2 discussed the McKinsey author team's value driver formula. An extended version of this discussion is now contained in a separate article, Jennergren 2013a.

²On the treatment of operating leases in connection with valuation, see Jennergren 2011. Pension contributions in McKay may be thought of as paid out to an outside pension fund concurrently with the salaries generating those contributions, so no pension debt remains on the company's books.

given below of net property, plant, and equipment (PPE) as driven by revenues agrees with Koller et al. 2010. The first two editions of *Valuation: Measuring and Managing the Value of Companies* contain two alternative model specifications relating to investment in PPE (cf. Levin and Olsson 1995).

In the following respect, this tutorial is an extension of Koller et al. 2010: It contains a more detailed discussion of capital expenditures, i. e., the mechanism whereby cash is absorbed by investments in PPE. This mechanism centers on two particular forecast assumptions, [this year's net PPE/revenues] and [depreciation/last year's net PPE].³ It is explained below how those assumptions can be specified consistently. On a related note, the treatment of deferred income taxes is somewhat different, and also more detailed, compared to Koller et al. 2010. In particular, deferred income taxes are related to a forecast ratio [timing differences/this year's net PPE], and it is suggested how to set that ratio.

There is also another extension in this tutorial: Alternative valuation models are discussed, in fact, five different models.

The McKay valuation is set up as a spreadsheet file in Excel named MCK.XLS. That file is an integral part of this tutorial. The model consists of the following parts, as can be seen by downloading the file and opening the first worksheet "Tables 1 - 8 and value calc":⁴

- Table 1. Historical income statements,
- Table 2. Historical balance sheets,
- Table 3. Historical free cash flow,
- Table 4. Historical ratios for forecast assumptions,
- Table 5. Forecasted income statements,
- Table 6. Forecasted balance sheets,
- Table 7. Forecasted free cash flow,
- Table 8. Forecast assumptions,
- Value calculations.

Tables in the spreadsheet file and in the file printout that is included in this tutorial are hence indicated by numerals, like Table 1. Tables in the tutorial text are indicated by capital letters, like Table A.

The outline of this tutorial is as follows: Section 2 gives an overview of essential model features. Section 3 summarizes the calculation of free cash flow. Section 4 is an introduction to forecasting financial statements and also discusses forecast assumptions

³Square brackets are used to indicate specific ratios that appear in sections in the spreadsheet files.

⁴All spreadsheet cells and rows in MCK.XLS that are mentioned in the sequel are located in the first worksheet "Tables 1 - 8 and value calc", except in Section 7 where there are some references to the second worksheet "Historical n and K ".

relating to operations and working capital. Sections 5, 6, and 7 deal with the specification of the forecast ratios [this year's net PPE/revenues], [depreciation/last year's net PPE], and [retirements/last year's net PPE]. Section 8 considers forecast assumptions about taxes. Further forecast assumptions, relating to discount rates and financing, are discussed in Section 9. Section 10 outlines the construction of forecasted financial statements and free cash flow, given that all forecast assumptions have been fixed. The discounting procedure is explained in Section 11. Section 12 gives results from a sensitivity analysis, i. e., computed values of McKay's equity when certain forecast assumptions are revised. Section 13 discusses valuations by two further models, the discounted dividends model and the adjusted present value model. A different way of calculating the WACC, without changing the financing policy, is discussed in Section 14, and valuations by the discounted cash flow and discounted dividends models with this new WACC are presented. All of the discussion so far refers to the file MCK.XLS. Section 15 considers a different financing policy for McKay. Under that financing policy, McKay is valued by five different models (economic value added and abnormal earnings, in addition to the three models already mentioned).⁵ Section 16 contains concluding remarks.

2 Model overview

Essential features of the discounted cash flow model are the following:

1. The model uses published accounting data as input. Historical income statements and balance sheets are used to derive certain critical financial ratios. Those historical ratios are used as a starting point in making predictions for the same ratios in future years.

2. The object of the discounted cash flow model is to value the *equity* of a going concern. Even so, the *asset side* of the balance sheet is initially valued. The value of the interest-bearing debt is then subtracted to get the value of the equity. Interest-bearing debt does not include deferred income taxes and trade credit (accounts payable and other current liabilities). Credit in the form of accounts payable is paid for not in interest but in higher operating expenses (i. e., higher purchase prices of raw materials) and is therefore part of operations rather than financing. Deferred income taxes are viewed as part of equity; cf. Sections 9 and 10. It may seem like an indirect approach to value the assets and deduct interest-bearing debt to arrive at the equity (i. e., it may seem more straight-forward to value the equity directly, by discounting future expected dividends). However, this indirect approach is the recommended one, since it leads to greater clarity and fewer errors in the valuation process (cf. Koller et al. 2010, pp. 102-103).

⁵See the file MCK_EXT.XLS. A printout from that file is also included here.

3. The value of the asset side is the value of operations plus *excess marketable securities*. The latter can usually be valued using book values or published market values. Excess marketable securities include cash that is not necessary for operations. For valuation purposes, the cash account may hence have to be divided into two parts, operating cash (which is used for facilitating transactions relating to actual operations), and excess cash. (In the case of McKay, excess marketable securities have been netted against interest-bearing debt at the date of valuation. Hence there are actually no excess marketable securities in the McKay valuation. This is one of the modelling choices that were alluded to in the introduction.)

4. The operations of the firm, i. e., the total asset side minus excess marketable securities, are valued by the WACC method. In other words, free cash flow from operations is discounted to a present value using the WACC. There is then a simultaneity problem concerning the WACC. More precisely, the debt and equity values enter into the WACC weights. However, equity value is what the model aims to determine.

5. The asset side valuation is done in two parts: Free cash flow from operations is forecasted for a number of individual years in the *explicit forecast period*. After that, there is a continuing (post-horizon) value derived from free cash flow in the first year of the *post-horizon period* (and hence individual yearly forecasts must be made for each year in the explicit forecast period and for one further year, the first one immediately following the explicit forecast period). The explicit forecast period should consist of at least 10 - 15 years (cf. Koller et al. 2010, p. 186). The explicit forecast period can be thought of as a transient phase during a turn-around or after a take-over. The post-horizon period, on the other hand, is characterized by steady-state development. This means that the explicit forecast period should as a minimal requirement be sufficiently long to capture transitory effects, e. g., during a turn-around operation. Actually, it is a requirement of the present implementation of the discounted cash flow model that the explicit forecast period should not be shorter than the economic life of the PPE.

6. For any future year, free cash flow from operations is calculated from forecasted income statements and balance sheets. This means that free cash flow is derived from a consistent scenario, defined by forecasted financial statements. This is probably the main strength of the discounted cash flow model, since it is difficult to make reasonable forecasts of free cash flow in a direct fashion. Financial statements are forecasted in nominal terms. This implies that nominal free cash flow is discounted using a nominal discount rate.

7. Continuing value is computed through an infinite discounting formula. In this tutorial, the Gordon (constant-growth) formula is used. In other words, free cash flow in the post-horizon period increases by some constant percentage from year to year, hence satisfying a necessary condition for infinite discounting. (The Gordon formula is another one of the modelling choices made in this tutorial.)

As can be inferred from this list of features, and as will be explained below, the discounted cash flow model combines three rather different tasks: The first one is the production of forecasted financial statements. This is not trivial. In particular, it involves issues relating to capital expenditures that are fairly complex. (The other valuation models use forecasted financial statements, just like the discounted cash flow model, so the first task is the same for those models as well.)

The second task is deriving free cash flow from operations from financial statements. At least in principle, this is rather trivial. In fairness, it is not always easy to calculate free cash flow from complicated historical income statements and balance sheets. However, all financial statements in this tutorial are very simple (and there is, in any case, no reason to *forecast* accounting complexities if the purpose is one of valuation). The third task is discounting forecasted free cash flow to a present value. While not exactly trivial, this task is nevertheless one that has been discussed extensively in the corporate finance literature, so there is guidance available. This tutorial will explain the mechanics of discounting in the discounted cash flow model. However, issues relating to how the relevant discount rates are determined will be treated only lightly. For more detailed discussions, the reader is referred to standard text books (for instance, Berk and DeMarzo 2011, chapter 18; Brealey et al. 2011, chapters 17 and 19; Ross et al. 2008, chapter 17).

3 Historical financial statements and the calculation of free cash flow

The valuation of McKay is as of Jan. 1 year 1. Historical input data are the income statements and balance sheets for the years -6 to 0 , Tables 1 and 2. Table 1 also includes statements of retained earnings. It may be noted in Table 1 that operating expenses do not include depreciation. In other words, the operating expenses are cash costs. At the bottom of Table 2, there are a couple of financial ratio calculations based on historical data for the given years. *Short-term debt* in the balance sheets (Table 2) is that portion of last year's long-term debt which matures within a year. It is clear from Tables 1 and 2 that McKay's financial statements are very simple, and consequently the forecasted statements will also have a simple structure. As already mentioned earlier, McKay has no excess marketable securities in the last historical balance sheet, i. e., at the date of valuation. A slightly puzzling feature of the historical financial statements may be noted: The relationship between interest income and excess marketable securities. That is, there is zero interest income in several years, even though excess marketable securities are positive.

From the data in Tables 1 and 2, historical free cash flow for the years -5 to 0 is computed in Table 3. Each annual free cash flow computation involves *two* balance

sheets, that of the present year and the previous one, so no free cash flow can be obtained for year -6 . Essentially the same operations are used to forecast free cash flow for year 1 and later years (in Table 7). The free cash flow calculations assume that the *clean surplus relationship* holds. This implies that the change in book equity (including retained earnings) equals net income *minus* net dividends (the latter could be negative, if there is an issue of common equity). The clean surplus relationship does not hold, if PPE is written down (or up) directly against common equity (for instance). Such accounting operations may complicate the calculation of free cash flow from historical financial statements (and if so, that calculation may not be trivial). However, there is usually no reason to forecast deviations from the clean surplus relationship in a valuation situation.

EBIT in Table 3 means Earnings Before Interest and Taxes. NOPLAT means Net Operating Profits Less Adjusted Taxes. *Taxes on EBIT* consist of calculated taxes according to the income statement (from Table 1) *plus* [this year's tax rate] \times (interest expense) *minus* [this year's tax rate] \times (interest income). Interest income and interest expense are taken from Table 1. The tax rate is given in Table 4. Calculated taxes according to the income statement reflect depreciation of PPE over the economic life. *Change in deferred income taxes* is this year's deferred income taxes *minus* last year's deferred income taxes. In the McKay valuation example, it is assumed that deferred income taxes come about for one reason only, *timing differences* in depreciation of PPE. That is, fiscal depreciation takes place over a period shorter than the economic life.⁶

Working capital is defined net. Hence, working capital consists of the following balance sheet items: Operating cash *plus* trade receivables *plus* other receivables *plus* inventories *plus* prepaid expenses *minus* accounts payable *minus* other current liabilities. Accounts payable and other current liabilities are apparently considered to be part of the operations of the firm, not part of the financing (they are not interest-bearing debt items). *Change in working capital* in Table 3 is hence this year's working capital *minus* last year's working capital. *Capital expenditures* are this year's net PPE *minus* last year's net PPE *plus* this year's depreciation. Depreciation is taken from Table 1, net PPE from Table 2. It should be emphasized that depreciation in Table 1 (and forecasted depreciation in Table 5) is according to plan, over the economic life of the PPE.

Free cash flow in Table 3 is hence cash generated by the operations of the firm, after paying taxes on operations only, and after expenditures for additional working capital and after capital expenditures. ("Additional working capital" could of course be negative. If so, free cash flow is *generated* rather than *absorbed* by working capital.) Hence, free cash flow represents cash that is available for distribution to the holders of debt and equity in the firm, and for investment in additional excess marketable securities. Stated somewhat

⁶Timing differences in depreciation of PPE have traditionally been a major reason for deferred income taxes. Cf. Koller et al. 2010, pp. 145-147 and 203, and Chapter 25, on deferred taxes.

differently, free cash flow is equal to *financial cash flow*, which is the utilization of free cash flow for financial purposes. Table 3 also includes a break-down of financial cash flow. By definition, free cash flow must be exactly equal to financial cash flow.

We now return briefly to the financial ratios at the end of Table 2. *Invested capital* is equal to working capital *plus* net PPE. Debt at the end of Table 2 in the ratio [debt/invested capital] is net interest-bearing (short-term and long-term debt minus excess marketable securities; the latter are hence counted as negative interest-bearing debt even though the interest income has been zero in some years). The financial ratio [NO-PLAT/invested capital] is also referred to as ROIC (Return on Invested Capital). It is a better analytical tool for understanding the company's performance than other return measures such as return on equity or return on assets, according to Koller et al. (2010, p. 164). Invested capital in the ratio [NOPLAT/invested capital] is the average of last year's and this year's. It is seen that McKay has provided a decreasing rate of return in recent years. It can also be seen from Table 3 that the free cash flow has been negative, and that the company has handled this situation by increasing its debt. It is evident from the bottom of Table 2 that the ratio of interest-bearing debt to invested capital has increased substantially from year -6 to year 0 .

Table 4 contains a set of historical financial ratios. Those ratios are important, since forecasts of the same ratios will be used to produce forecasted income statements and balance sheets. Most of the items in Table 4 are self-explanatory, but a few observations are called for. Net PPE (which is taken from Table 2) enters into four ratios. In two of those cases, [depreciation/net PPE] and [retirements/net PPE], the net PPE in question is last year's. In the other two cases, [net PPE/revenues] and [timing differences/net PPE], the net PPE in question is this year's. *Retirements* are defined as depreciation *minus* change in accumulated depreciation between this year and last year (accumulated depreciation is taken from Table 2). This must hold, since last year's accumulated depreciation *plus* this year's depreciation *minus* this year's retirements equals this year's accumulated depreciation.

The *timing differences* for a given year are measured between accumulated fiscal depreciation of PPE and accumulated depreciation according to PPE economic life. For a given piece of PPE that is about to be retired, accumulated fiscal depreciation and accumulated depreciation according to economic life are both equal to the original acquisition value. Consequently, non-zero timing differences are related to non-retired PPE only. The ratio [timing differences/net PPE] in Table 4 has been calculated by first dividing the deferred income taxes for a given year by the same year's corporate tax rate (also given in Table 4). This gives that year's timing differences. After that, there is a second division by that year's net PPE.

4 Forecast assumptions relating to operations and working capital

Having recorded the historical performance of McKay in Tables 1 - 4, we now turn to the task of forecasting free cash flow for years 1 and later. Individual free cash flow forecasts are produced for each year 1 to 12. The free cash flow amounts for years 1 to 11 are discounted individually to a present value. The free cash flow for year 12 and all later years is discounted through the Gordon formula, with the free cash flow in year 12 as a starting value. Years 1 to 11 are therefore the explicit forecast period, and year 12 and all later years the post-horizon period. As required, the explicit forecast period is at least as long as the economic life of the PPE (the latter is assumed to be 10 years in Section 7 and 11 years in a sensitivity analysis scenario in Section 12).

Tables 5 - 8 have the same format as Tables 1 - 4. In fact, Table 5 may be seen as a continuation of Table 1, Table 6 as a continuation of Table 2, and so on. We start the forecasting job by setting up Table 8, the forecast assumptions. Using assumptions (financial ratios and others) in that table, and using a couple of further direct forecasts of individual items, we can set up the forecasted income statements, Table 5, and the forecasted balance sheets, Table 6. From Tables 5 and 6, we can then in Table 7 derive the forecasted free cash flow (just like we derived the historical free cash flow in Table 3, using information in Tables 1 and 2).

Consider now the individual items in Table 8. It should be noted in Table 8 that all items are the same for year 12, the first year of the post-horizon period, as for year 11, the last year of the explicit forecast period. Since the first year in the post-horizon period is representative of all subsequent post-horizon period years, all items are the same for every post-horizon period year as for the last year of the explicit forecast period. This is actually an important condition (cf. Levin and Olsson 1995, p. 38; Lundholm and O’Keefe 2001, pp. 321-322): If that condition holds, then free cash flow increases by the same percentage (the nominal revenue growth rate for year 12 in Table 8, cell T135) between all successive years in the post-horizon period. This means that a necessary condition for discounting by means of the Gordon formula in the post-horizon period is satisfied.

The *revenue growth* in each future year is a combination of inflation and real growth. More precisely, nominal revenue growth is “one plus real growth multiplied by one plus expected inflation minus one”. Actually, in years 10 and 11 there is no real growth, and the same assumption holds for all later years as well. The underlying assumption in Table 8 is apparently that real operations will initially expand but will eventually (in year 10) settle down to a steady state with no further real growth. Inflation, on the other hand, is assumed to be 3% in all coming years (including after year 12). This means, in particular, that the nominal revenue growth rate in the post-horizon period, which is used in the

Gordon formula, is 3%. The ratio of *operating expenses to revenues* is assumed to improve immediately, e. g., as a consequence of a determined turn-around effort. Apparently, it is set to 90% year 1 and all later years. To avoid misunderstandings, this forecast assumption and the other ones displayed in Table 8 are not necessarily intended to be the most realistic ones that can be imagined. The purpose is merely to demonstrate the mechanics of the discounted cash flow model for one particular scenario. A table in Levin and Olsson 1995 (p. 124; based on accounting data from Statistics Sweden) contains information about typical values of the ratio between operating expenses and revenues in various Swedish industries (cf. also Jennergren 2008, pp. 1554-1555, for a further discussion of the Statistics Sweden data base).

A number of items in the forecasted income statements and balance sheets are directly driven by revenues. That is, those items are forecasted as percentages of revenues. In particular, this holds for most of the *working capital* items. The idea is that as revenues increase, the required amounts of working capital also increase. It is not important whether revenues increase due to inflation or real growth, or a combination of both. Working capital turns over very quickly, and therefore it is a reasonable assumption that these working capital items are proportional to revenues. The ratios between the different working capital items and revenues in the first explicit forecast period year in Table 8 have been set equal to the average historical values. It is assumed that there will be an improvement (decrease) in these ratios for working capital *assets* over the explicit forecast period of 3% of the historical ratios (see cell S140), with linear interpolation in between the first and last explicit forecast period years. In other words, these ratios in the last explicit forecast period year are equal to 0.97 times the same ratios in the first explicit forecast period year. There is no corresponding change for the working capital *liabilities* (i. e., these ratios are all equal to the historical averages, meaning that the company resists pressure for faster payment from outside suppliers). Two of the working capital items, inventories and accounts payable, are forecasted as percentages of operating expenses rather than as percentages of revenues. This is actually not a very important distinction (i. e., one may perhaps just as well forecast all working capital items as percentages of revenues; cf. Koller et al. 2010, pp. 199-200). Again, these assumptions as regards working capital are only for illustrative purposes. Another table in Levin and Olsson 1995 (p. 125), again based on data from Statistics Sweden, reports average values of the ratio between (aggregate) working capital and revenues in different Swedish industries.

5 Forecast assumptions relating to property, plant, and equipment

The forecast assumptions relating to PPE will be considered next (this section and the following two). The equations that determine capital expenditures may be stated as follows (subscripts denote years):

$$\begin{aligned}(\text{capital expenditures})_t &= (\text{net PPE})_t - (\text{net PPE})_{t-1} + \text{depreciation}_t, \\(\text{net PPE})_t &= \text{revenues}_t \times [\text{this year's net PPE/revenues}], \\ \text{depreciation}_t &= (\text{net PPE})_{t-1} \times [\text{depreciation/last year's net PPE}].\end{aligned}$$

To this set of equations, we may add three more that are actually not necessary for the model:

$$\begin{aligned}\text{retirements}_t &= (\text{net PPE})_{t-1} \times [\text{retirements/last year's net PPE}], \\(\text{accumulated depreciation})_t & \\ &= (\text{accumulated depreciation})_{t-1} + \text{depreciation}_t - \text{retirements}_t, \\(\text{gross PPE})_t &= (\text{net PPE})_t + (\text{accumulated depreciation})_t.\end{aligned}$$

In particular, this second set of three equations is needed only if one wants to produce forecasted balance sheets showing how net PPE is equal to gross PPE minus accumulated depreciation. It should be noted that such detail is not necessary, since the first set of three equations suffices for determining net PPE, depreciation, and consequently also capital expenditures.⁷

It is clear from the first three equations that forecasts have to be made for two particular ratios, [this year's net PPE/revenues] and [depreciation/last year's net PPE]. Setting those ratios in a consistent fashion involves somewhat technical considerations. In this section and the following one, one way of proceeding, consistent with the idea of the company developing in a *steady-state* fashion in the post-horizon period, will be outlined.

To begin with, the idea of the company developing in a steady-state fashion has to be made more precise. As indicated in Section 4, the forecast assumptions should be specified in such a manner that nominal free cash flow increases by a constant percentage every year in the post-horizon period. This is a necessary condition for infinite discounting by the Gordon formula. But if so, capital expenditures must also increase by the same constant percentage in every post-horizon period year. For this condition on capital expenditures to hold, there must be an *even age distribution* of nominal acquisition values

⁷If the historical financial statements do not show gross PPE and accumulated depreciation, only net PPE, then it seems pointless to try to include these items in the forecasted financial statements. If so, the second set of three equations is deleted. In the McKay case, the historical statements do indicate gross PPE and accumulated depreciation. For that (aesthetic) reason, those items will also be included in the forecasted statements.

of successive PPE cohorts. More precisely, it must hold that the acquisition value of each PPE cohort develops in line with the assumed constant growth percentage that is applicable to the post-horizon period. That constant percentage is the same as the assumed nominal revenue growth in the post-horizon period, 3% in the McKay example, as was mentioned in Section 4.

The general idea is now to set steady-state values of the two ratios [this year's net PPE/revenues] and [depreciation/last year's net PPE] for *the last year of the explicit forecast period* (year 11 in the McKay example). Those steady-state values will then also hold for every year in the post-horizon period (since all forecast assumptions have to be the same in the first year of the post-horizon period as in the last year of the explicit forecast period, as already explained in Section 4).

During the preceding years of the explicit forecast period, steady-state values of [this year's net PPE/revenues] and [depreciation/last year's net PPE] are not assumed. Values for these two ratios in the preceding explicit forecast period years are fixed in the following heuristic fashion in the McKay example: For the first year of the explicit forecast period, they are set as averages of the corresponding values for the historical years.⁸ Values for intermediate (between the first and last) years in the explicit forecast period are then determined by interpolation.⁹

6 The ratios [this year's net PPE/revenues] and [depreciation/last year's net PPE] in the last year of the explicit forecast period

It is helpful at this point to proceed more formally and introduce the following notation:

g real growth rate in the last year of the explicit forecast period and in the

⁸The value for the last year of the explicit forecast period of [retirements/last year's net PPE] is also set as a steady-state value. For the first year of the explicit forecast period, that ratio is set equal to the corresponding value for the last historical year. An average of corresponding values for all historical years is not used in this case, since [retirements/last year's net PPE] appears to have been unstable during years -5 to 0. The negative value of that ratio in year -2 could have come about through purchases of used (second-hand) PPE. It is again noted that the ratio [retirements/last year's net PPE] is actually not necessary for the valuation model. It may also be noted that gross PPE in year 11 in cell S46 (645.6) is very close to the sum of capital expenditures in row 103 over the years 2 - 11, i. e., over the assumed economic life n which is 10 years.

⁹In Jennergren 2010, the ratios [this year's net PPE/revenues] and [depreciation/last year's net PPE] are actually set in a slightly different fashion, by interpolation between the average historical values and the steady-state values also in the first explicit forecast period year. In other words, these ratios are not set equal to the average historical values in the first explicit forecast period year.

	post-horizon period,
i	inflation rate in the last year of the explicit forecast period and in the post-horizon period,
c	nominal (composite) growth rate = $(1 + g)(1 + i) - 1$,
n	economic life of PPE (assumed to be integer years),
q	life of PPE for fiscal depreciation; see Section 8 (assumed to be integer years),
K	required <i>real gross</i> PPE divided by (real) revenues in the last year of the explicit forecast period and in the post-horizon period,
M	ratio between this year's <i>nominal gross</i> PPE and (nominal) revenues in the last year of the explicit forecast period and in the post-horizon period,
F_g	backwards summation factor expressing <i>real gross</i> PPE,
F_c	backwards summation factor expressing <i>nominal gross</i> PPE,
a	acquisition value of last PPE cohort (nominal <i>and</i> real; real = nominal <i>now</i>),
H	steady-state accumulated depreciation as a fraction of gross PPE,
J	factor expressing timing differences; see Section 8.

It is assumed in this tutorial that g and i are non-negative. To assume negative inflation over an infinite number of years is simply not credible. Negative real growth of the firm over an infinite number of years is also not realistic in connection with the discounted cash flow model. If such a situation were really foreseen, then a *break-up valuation* would be more relevant than a going concern valuation (as implied by the discounted cash flow model). Apparently, in the McKay example $g = 0\%$, $i = 3\%$, and consequently $c = 3\%$ in the last year of the explicit forecast period and from then on.

The main task in this section is to set the steady-state value of the ratio [this year's net PPE/revenues]. As before, by steady state is meant that the acquisition values of successive PPE cohorts increase by c , the nominal growth rate of revenues. Also as noted before, steady-state values of all forecast ratios must be attained already in the last year of the explicit forecast period.

At this point, there is a need for some model of the relationship between revenues and PPE, that is, a model of the firm's production. It is assumed here that revenues are related to *real gross* PPE through a capital intensity parameter K . In other words, in the last year of the explicit forecast period and from then on, real gross PPE must be equal to revenues multiplied by K . *Real* means expressed in the value of money of the current year in question. Real revenues are equal to nominal revenues for the current year. Real gross PPE means nominal gross PPE adjusted for inflation. Such an adjustment implies revaluing each PPE cohort, through multiplication by a factor that expresses accumulated inflation since that cohort was acquired. By relating revenues to *real* gross PPE, one eliminates effects due to inflation. The assumption that revenues are related to *gross* rather than *net* PPE implies that each piece of PPE is 100% productive

until the end of its economic life. At that point in time, it suddenly ceases to function and is retired. This seems like a somewhat more intuitive hypothesis than the alternative, relating revenues to *net* PPE, since that would mean that the productivity of each piece of PPE is proportional to its remaining economic life.

It is the steady-state value of the ratio [this year's *net* PPE/revenues] that is the object here, but initially M will be derived, that is, the ratio between this year's nominal *gross* PPE and (nominal) revenues in the last year of the explicit forecast period and in the post-horizon period. After that, M is multiplied by a factor $(1 - H)$ expressing steady-state *net* PPE as a fraction of steady-state gross PPE, hence providing steady-state [this year's *net* PPE/revenues].

Suppose now that a is the acquisition value of the last PPE cohort, which has just been purchased at the end of the current year. That acquisition value is the *real* one, expressed in current monetary units. Given the steady-state assumption, which implies that the acquisition values of previous cohorts have increased in *real* terms by the real growth rate g from year to year, the *real* value of gross PPE (in current monetary units and at the end of the current year) is hence $F_g \cdot a$, where¹⁰

$$F_g = \sum_{v=0}^{n-1} \left(\frac{1}{1+g} \right)^v = \frac{1+g - (1+g)^{-(n-1)}}{g} \text{ if } g > 0; \quad F_g = n \text{ if } g = 0.$$

The physical requirement for gross PPE then implies that

$$F_g \cdot a = K \cdot \text{revenues}.$$

Similarly, the *nominal* value of gross PPE at the end of the current year, under the steady-state assumption, is $F_c \cdot a$, where

$$F_c = \sum_{v=0}^{n-1} \left(\frac{1}{1+c} \right)^v = \frac{1+c - (1+c)^{-(n-1)}}{c} \text{ if } c > 0; \quad F_c = n \text{ if } c = 0.$$

Consequently,

$$F_c \cdot a = M \cdot \text{revenues}.$$

The formulas for F_g and F_c are contained in cells S153 and S154 in Table 8. It follows that (cell S156)

$$M = (F_c/F_g) \cdot K.$$

¹⁰The formulas for F_g and F_c use the summation

$$\sum_{v=0}^{\omega} x^v = \frac{1-x^{\omega+1}}{1-x} \quad (x \neq 1).$$

The following summation formula is also used below

$$\sum_{v=0}^{\omega} x^v v = \frac{d}{dx} \left(\sum_{v=0}^{\omega} x^v \right) \cdot x = \frac{-(\omega+1)x^{\omega}(1-x) + (1-x^{\omega+1})}{(1-x)^2} \cdot x \quad (x \neq 1). \quad (1)$$

Accumulated depreciation as a fraction of gross PPE in a steady state, H , can be written as (using (1) with $\omega = n - 1$; cf. also Levin and Olsson 1995, pp. 37, 51):

$$H = \frac{\sum_{v=0}^{n-1} \left[\left(\frac{1}{1+c} \right)^v \cdot \frac{v}{n} \right]}{F_c} = \frac{-n(1+c)^{-(n-1)}(1-(1+c)^{-1}) + (1-(1+c)^{-n})}{(1-(1+c)^{-1})^2} \cdot \frac{1}{1+c} \cdot \frac{1}{n} =$$

$$\frac{1+c-(nc+1)(1+c)^{-(n-1)}}{c^2 n} = \frac{1}{cn} - \frac{1}{(1+c)^n - 1} \text{ if } c > 0; \quad H = \frac{n-1}{2n} \text{ if } c = 0. \quad (2)$$

The formula for H is contained in cell S157. The desired steady-state ratio [this year's net PPE/revenues] is then

$$M(1 - H). \quad (3)$$

This is the formula in cell S158 of Table 8.

Assuming linear depreciation over the economic life of the PPE, the steady-state ratio [depreciation/last year's net PPE] is

$$\frac{1}{n} \cdot \frac{1}{1 - H}. \quad (4)$$

This is the formula in cell S159 of Table 8.¹¹

The steady-state ratios derived in this section apparently depend on four parameters, the real growth rate g , the inflation rate i (since c depends on g and i), the capital intensity K , and the economic life n of the PPE.¹² Armed with the formulas derived here, one can perform sensitivity analyses of how calculated equity value varies due to changes in these four parameters.

7 On the implementation of assumptions relating to PPE

The forecast for the ratio [this year's net PPE/revenues] in the last year of the explicit forecast period can hence be obtained as equation (3) in the previous section, given that

¹¹The steady-state formula for [retirements/last year's net PPE] is

$$\frac{(1+c)^{-n}}{F_c(1+c)^{-1}} \cdot \frac{1}{1-H} = \frac{1}{F_c(1+c)^{(n-1)}} \cdot \frac{1}{1-H}.$$

This is the formula in cell S160 in Table 8.

¹²Actually, steady-state [depreciation/last year's net PPE] and steady-state [retirements/last year's net PPE] depend on two parameters only, c and n . That is, they do not depend on g and i separately. All that matters for these two ratios is nominal growth c , not how that growth comes about due to different combinations of real growth g and expected inflation i .

g , i , n , and K have been specified. The specification of n and K is not self-evident, so here are some suggestions on how to estimate these two parameters from historical data.

The following discussion refers to the second worksheet “Historical n and K ” of the file MCK.XLS. Consider first estimating n from the company’s own historical financial statements. A simple way of estimating n is to take an average of historical (depreciation/last year’s gross PPE). This is only feasible if the historical financial statements show gross PPE and accumulated depreciation in addition to net PPE. That is the case for McKay, and the estimate is calculated in rows 17 - 20. It is apparently equal to 10.6. Since n should be integer (years), one would round to 11.

There is an alternative way, shown in rows 22 - 32, to estimate n that does not require information about historical gross PPE. It uses the steady-state formula (4) for [depreciation/last year’s net PPE] and applies that formula to the observed values of that ratio in the historical years. More precisely, suppose one takes the average of all available historical observations of [depreciation $_t$ /(net PPE) $_{t-1}$]. There are apparently 6 such observations. Proceeding as indicated,

$$\frac{\sum[\text{depreciation}_t/(\text{net PPE})_{t-1}]}{6} = \frac{1}{n} \cdot \frac{1}{1-H}. \quad (5)$$

The left hand side of (5) is calculated in cell H26. The right hand side, formula (4), depends only on c and n . The average historical nominal growth is estimated as 17.0% in cell H15. With that value of c , one can search for that integer value of n for which the right hand side of (5) is the closest to the left hand side. That value of n is 10 (cell H27). This calculation is not intended to suggest that the company was in steady state in the historical years. Rather, (5) is merely a heuristic device for an approximate estimate of n .

There are now two suggested values of n , 11 which is obtained by rounding 10.6 in cell H20, and 10 in cell H27. The value 10 is selected. This is the n value that is assumed for the last year of the explicit forecast period in cell S152 in the first worksheet “Tables 1 - 8 and value calc”. Actually, $n = 11$ is also used in a sensitivity analysis scenario in Section 12 below.

Next, we want to estimate K from the company’s historical financial statements. Here is a first way. For each one of the last n historical years, one determines the capital expenditures, like in Table 3. Apparently, this means that $n + 1$ sets of historical financial statements must be available. Each such amount except the last one is then inflated to the price level that is valid for the last historical year. This is done using some suitable time series of historical inflation rates during the $n - 1$ preceding historical years. After that, all n amounts are summed, and the sum is divided by revenues in the last historical year. In the McKay example, this procedure is not applicable, however, since $n + 1 = 11$ sets of historical financial statements are not available (financial statements are available only for 7 historical years).

Fortunately, there is an alternative way to estimate K that works with fewer years of historical financial statements (rows 34 - 42). Taking the average of all available historical observations (7 observations) of $[(\text{net PPE})_t/\text{revenues}_t]$ and using the steady-state formula (3) for [this year's net PPE/revenues],

$$\frac{\sum[\text{net PPE})_t/\text{revenues}_t]}{7} = M(1 - H). \quad (6)$$

The left hand side of (6) is calculated in cell H37. The right hand side, formula (3), depends on g , $c = (1 + g)(1 + i) - 1$, n , and K . n has already been estimated to 10, and the average nominal growth c to 17.0%. Under the assumption that historical inflation i has been equal to 3% (row 12), average historical real growth g is estimated as 13.6% in cell H14. With these values of g , c , and n , (6) can be solved for K . This is done in cell H42. The result is 0.581. The value for K that is assumed for the last year of the explicit forecast period in cell S155 of the first worksheet "Tables 1 - 8 and value calc" is 0.580.¹³ In other words, it is assumed that the capital intensity at the horizon is roughly the same as the capital intensity that is estimated from the historical data. Again, it is not suggested here that the company was in a steady state in the historical years. Rather, (6) is merely a heuristic device for estimating K . – This ends the discussion of the worksheet "Historical n and K ".

An even more heuristic approach would be to set K so as to obtain a "reasonable" value of the ratio [this year's net PPE/revenues] in the last year of the explicit forecast period, reasonable meaning in relation to what has actually been observed in historical years. It is assumed here that g , i , and n have already been fixed. That is, K is set after these other three. Under this approach, there is no attempt to ascertain what K has actually been in the historical period. One merely uses K as a free parameter to obtain a forecasted value of the ratio [this year's net PPE/revenues] in the last year of the explicit forecast period that seems acceptable.

Another approach to setting n and K is to take as a starting point the data base from Statistics Sweden that was mentioned in Section 4. It is indicated in Jennergren 2008, pp. 1554-1555, how that data base can be used to provide rough estimates of n and K . The calculations are very similar to the calculations earlier in this section of n and K from the company's historical financial statements. A table on p. 1555 of Jennergren 2008 contains suggested values for various industries. It has been noted in a number of valuation projects, though, that the K values in that table often appear rather high. For instance, K is equal to 0.81 for the land transportation industry (using data pertaining to 1994 - 1998). But that is much too high for the McKay example, even though it refers to a trucking company, and hence to the land transportation industry. One reason why

¹³The estimated K value apparently changes very little in response to changes in the assumed n . In the sensitivity analysis scenario with $n = 11$ in Section 12, K therefore remains at 0.580.

it is too high could be that land transportation also includes railways, i. e., more capital intensive activities than trucking.

The McKay example considers only one homogeneous category of PPE with an assumed economic life of $n = 10$ years, as already mentioned above. One can of course set up a valuation model with different categories of PPE, e. g., machinery and buildings. The economic life of each category is usually mentioned in company annual reports. The assumption that n is integer is not restrictive, if different categories of PPE are considered, since individual categories can be thought of as having different integer economic lives.

To recapitulate, this section and the previous two have considered forecasts for three particular ratios, [this year's net PPE/revenues], [depreciation/last year's net PPE], and [retirements/last year's net PPE]. Steady-state values of these ratios can be specified for the last year of the explicit forecast period. Those steady-state values depend on real growth g , inflation i , PPE economic life n , and capital intensity K (required real gross PPE divided by revenues). They are consistent with the company developing in a steady-state fashion in the post-horizon period, and consequently with the general idea of dividing the future into explicit forecast and post-horizon periods. However, there is not total consistency, for (at least) two reasons. In the first place, the steady-state assumption is obviously only an approximation: Successive PPE cohorts when entering the post-horizon period, as resulting from capital expenditures in the explicit forecast period, cannot be expected to satisfy precisely the even age distribution requirement. This inconsistency is usually not very important. In the second place, real gross PPE when entering the post-horizon period (again the result of forecasted capital expenditures in the explicit forecast period) does not automatically correspond exactly to what is needed according to the capital intensity parameter K (i. e., $K \cdot$ revenues).

For the earlier years in the explicit forecast period, [this year's net PPE/revenues], [depreciation/last year's net PPE] and [retirements/last year's net PPE] have been set in a heuristic fashion in the McKay example (see Table 8): Values for the first year of the explicit forecast period have been set equal to the average of all corresponding historical ratios, or equal to the immediately preceding historical ratio. Values for intermediate years of the ratio [this year's net PPE/revenues] have been determined by *non-linear* interpolation between the first and last years of the explicit forecast period, in such a manner that the real gross PPE when entering the post-horizon period is equal to what is required according to the capital intensity K .¹⁴ Hence, the second inconsis-

¹⁴See rows 161 - 167! Cell S164 calculates actual gross PPE at the end of the last explicit forecast period year, as resulting from capital expenditures that have been undertaken in that year and the $n - 1$ previous explicit forecast period years. This calculation uses inflation factors in row 163. The amount of real gross PPE that is needed is given in cell S165. The difference between cells S164 and S165, multiplied by 1,000,000, is contained in cell I166. This difference can be driven to zero by adjusting the interpolation

tency in the previous paragraph is eliminated. Values for intermediate years of the ratios [depreciation/last year's net PPE] and [retirements/last year's net PPE] are determined through linear interpolation. This is an easy way of making forecasts for the intermediate years of the explicit forecast period. It is proposed here as a simple-minded alternative to bottom-up forecasting of individual capital expenditures (new and replacement). The latter alternative may be more accurate but is also more complex, since it can usually only be done using information available inside a company, i. e., not on the basis of published accounting data (cf. Koller et al. 2010, pp. 194-195; cf. also Jennergren 2010 for a more extensive discussion of the forecasting of PPE and depreciation in the explicit forecast period).

8 Forecast assumptions relating to taxes

The next set of forecast assumptions in Table 8 refers to taxes. The corporate tax rate has apparently been 39% in all historical years and is forecasted to remain at that level in the future. The further tax assumption that must be fixed for future years is the ratio [timing differences/this year's net PPE]. This ratio relates to the balance sheet item deferred income taxes. That is, deferred income taxes are equal to (this year's net PPE) \times [timing differences/this year's net PPE] \times [this year's tax rate]. It may be noted that deferred income taxes are revalued when the tax rate changes (the so-called liability method of accounting for deferred taxes). The precise steps of that revaluation will be mentioned in Section 10 below. In the base case McKay scenario, there is actually no need for such a revaluation, since the tax rate is the same in all historical and future years. However, in a sensitivity analysis one may wish to assume a different tax rate for future years, e. g., starting with year 1 (cf. Scenario 8 in Section 12 below). If so, there will be an error in the free cash flow calculation, unless deferred income taxes are revalued.

The ratio [timing differences/this year's net PPE] can be set in the same fashion as in the previous three sections. That is, a value for the first year of the explicit forecast period is set as an average of the corresponding historical values. A value for the last year of the explicit forecast period is specified through steady-state considerations, like the values for the ratios relating to PPE. Values for intermediate years are then fixed by linear interpolation. This procedure has been followed in the McKay example.

As already indicated in Section 6, the life of the PPE for depreciation for tax purposes is denoted by q . It is obviously assumed that $q \leq n$. Also, it is assumed that each piece

curve parameter α in cell S162. $\alpha = 0$ means no curvature, i. e., linear interpolation. Finding that value of α that gives a difference of zero in cell I166 can conveniently be done using the Goal Seek procedure. This non-linear interpolation procedure obviously presupposes that the explicit forecast period comprises at least n years, i. e., as many years as the economic life of the PPE.

of PPE is depreciated linearly for tax purposes, i. e., by $1/q$ of the acquisition value each year.

If the steady-state condition holds, i. e., the acquisition values of successive PPE cohorts increase by c , then the ratio [timing differences/this year's net PPE] in the last year of the explicit forecast period can be written as

$$\frac{J}{F_c(1-H)}, \quad (7)$$

where

$$\begin{aligned} J &= \sum_{v=0}^{q-1} \left(\left(\frac{1}{1+c} \right)^v \cdot \frac{v}{q} \right) + \sum_{v=q}^{n-1} \left(\frac{1}{1+c} \right)^v - \sum_{v=0}^{n-1} \left(\left(\frac{1}{1+c} \right)^v \cdot \frac{v}{n} \right) \\ &= \frac{1+c - (qc+1)(1+c)^{-(q-1)}}{c^2q} + \frac{1+c - (1+c)^{-(n-q-1)}}{c} \cdot \frac{1}{(1+c)^q} \\ &\quad - \frac{1+c - (nc+1)(1+c)^{-(n-1)}}{c^2n} \end{aligned}$$

if $c > 0$. The first term in J represents accumulated fiscal depreciation for PPE cohorts that have not yet been written down to zero for tax purposes, the second term accumulated fiscal depreciation for those PPE cohorts that have already been written down to zero for tax purposes but have not yet been retired, and the third term accumulated depreciation over the economic lives for PPE cohorts that have not yet been retired. (Cf. the remark at the end of Section 3 to the effect that non-zero timing differences are related to non-retired PPE cohorts only; cf. also equation (2) in Section 6 for part of the derivation.) If $c = 0$, then

$$J = 0.5(q-1) + (n-q) - 0.5(n-1).$$

The formula for J is contained in cell S172 in Table 8. Equation (7), the steady-state ratio [timing differences/this year's net PPE] in the last year of the explicit forecast period, is contained in cell S173.

9 Forecast assumptions relating to discount rates and financing

Consider now the interest rate items in Table 8. The nominal borrowing rate is “one plus the real rate multiplied by one plus expected inflation minus one”. McKay's real borrowing rate is apparently forecasted to be 5.60% in all future years. Expected inflation has already earlier been forecasted to remain at 3% in future years. The nominal

borrowing rate is hence $(1+0.0560) \times (1+0.03) - 1 = 8.77\%$ (rounded).¹⁵ Incidentally, the forecasted nominal borrowing rate is assumed to be the going market rate for companies in McKay's risk class. This means that *the market value of the interest-bearing debt is equal to the book value*. In the valuation of the equity as a residual, the book value of the interest-bearing debt is subtracted from the value of the firm's assets. This procedure is correct only because of the equality between market and book debt values when the nominal borrowing rate is the same as the going market rate (cf. Jennergren 2005 on debt valuation when this assumption does not hold).

For calculating the WACC, the cost of equity capital, also referred to as the required rate of return on equity, is also needed. The real cost of equity capital is apparently assumed to be 11.40%. The nominal cost of equity capital then becomes $(1 + 0.1140) \times (1+0.03) - 1 = 14.74\%$ (rounded). It should be emphasized that the cost of equity capital, as well as the borrowing rate, is *not independent* of the debt and equity weights that enter into the WACC. In fact, the nominal borrowing rate in row 177 and the nominal cost of equity in row 179 are valid under the assumption that the WACC weights are 50% debt and 50% equity.¹⁶ If those debt and equity weights are varied, then the borrowing rate and cost of equity capital should be varied as well. However, the precise relationship between, on the one hand, the debt and equity weights entering into the WACC and, on the other hand, the borrowing rate and cost of equity capital that also enter into the WACC is for the time being (until Section 14 below) left unspecified in this tutorial. Hence, there is not much explicit modelling of the borrowing rate and cost of equity capital in Table 8. It should be noted, though, that both of these interest rate items depend on assumed inflation. If inflation increases, then so do the nominal borrowing rate and nominal cost of equity capital.

The next-to-last item in Table 8 is [book value target for financial strength]. Financial strength is defined as (invested capital *minus* interest-bearing debt) *divided by* invested capital (it is recalled from Section 3 that invested capital equals working capital *plus* net PPE). This ratio apparently refers to McKay's financing policy. The financing policy is the means to guarantee that there will be an equality between the assets and liabilities sides of the forecasted balance sheets. More precisely, total common equity or interest-bearing

¹⁵It is assumed that the *before-tax* real borrowing rate remains constant under varying inflation expectations. A different relationship between the nominal borrowing rate and expected inflation is obtained, if one assumes that it is the *after-tax* real borrowing rate that stays constant under varying inflation expectations. See Howe 1992 for a discussion of this issue. The assumption made here, that the before-tax real borrowing rate remains constant as inflation expectations change, seems to agree with empirical findings (Howe 1992, p. 34).

¹⁶As will be seen below (Section 11), those weights are applicable to a *target capital structure in market value terms in the first year of the post-horizon period*. The same weights are then applied in all of the years of the explicit forecast period, and in all later years of the post-horizon period.

debt must be determined as the residual.

The following financing policy has been assumed for McKay: The company's recent performance has been rather shaky, as evidenced by the fact that the ratio [debt/invested capital] at the bottom of Table 2 has increased substantially. McKay should try to reduce that ratio and hence improve its financial strength over the coming years (as viewed from the date of valuation, Jan. 1 of year 1). For that purpose, no dividends will be paid at all, as long as financial strength is below the target in row 183 of Table 8. Otherwise, maximal dividends are paid out, while still keeping financial strength as required. Obviously, this is only intended as one example of a financing policy that can be incorporated into the discounted cash flow model. A *book value* target for financial strength can conveniently be adjusted to provide a target capital structure *in market value terms* in the first year of the post-horizon period.¹⁷

Consequently, there is a ratio [book value target for financial strength]. Borrowing as well as dividends are adjusted to reach that target (however, negative dividends are not allowed). Deferred income taxes are viewed as part of equity in the discounted cash flow model (cf. also Brealey et al. 2011, p. 507; Koller et al. 2010, pp. 145-146). Hence, deferred income taxes are not subtracted in the calculation of equity value as a residual. McKay's [book value target for financial strength] in row 183 in Table 8 can therefore be restated as follows: The sum of the three items deferred income taxes, common stock, and retained earnings on the liabilities side of the balance sheet should equal 55.8% of invested capital. Equivalently, interest-bearing debt should be 44.2% of invested capital. Apparently, the assumption is that [book value target for financial strength] should be the same each year.

The financial structure of the firm, including the dividend policy, does not affect the computed free cash flow. The financial structure does affect the *valuation* of free cash flow, though, through the WACC computation.¹⁸

The final item in Table 8 is [this year's short-term interest-bearing debt/last year's long-term interest-bearing debt]. This ratio only serves to divide total interest-bearing debt in the forecasted balance sheets into short-term and long-term. It does not have any effect on the valuation in the McKay example, since the nominal borrowing rate does not depend on loan contract length.

There are no further assumptions for forecasting income statements and balance sheets in Table 8. However, two additional assumptions have been incorporated directly into the

¹⁷In fact, the ratio [book value target for financial strength] 55.8% mentioned below has been selected so as to reach a target capital structure in market value terms in year 12 of 50% equity and 50% debt (cf. Section 11).

¹⁸Financial structure may affect computed free cash flow in more complex situations, for instance if the company has tax-loss carry-forwards.

forecasted financial statements, i. e., not by way of ratios in Table 8. It is directly assumed that there will be no new issue of equity (i. e., the item common stock in the balance sheets remains at the same level as in the last historical year). Also, the excess marketable securities are assumed to remain at zero in all forecasted balance sheets.

10 Forecasted income statements, balance sheets, and free cash flow

With the forecast assumptions in Table 8 and the additional assumptions that were noted in the previous section, we can now construct the forecasted income statements in Table 5 and forecasted balance sheets in Table 6 for years 1 to 12. Revenues in Table 5 are (last year's revenues) \times (1 plus [revenue growth]) ([revenue growth] is taken from Table 8). Operating expenses are revenues multiplied by [operating expenses/revenues] (also from Table 8). Depreciation in Table 5 is last year's net PPE multiplied by [depreciation/last year's net PPE] (from Table 8). Interest income is equal to last year's excess marketable securities multiplied by the nominal borrowing rate (i. e., McKay is assumed to earn the borrowing rate on its excess cash; however, in this case the result is zero since the excess marketable securities are set to zero in the last historical year and in all future years). Interest expense is the assumed nominal borrowing rate (from Table 8) applied to the sum of last year's short-term and long-term debt.

The item revaluation of deferred income taxes in Table 5 is obtained by recomputing last year's deferred income taxes in accordance with this year's tax rate and subtracting the result from last year's deferred income taxes as stated in last year's balance sheet. The recomputation part consists of dividing last year's deferred income taxes by last year's tax rate (from Table 4 when the last year is year 0 and otherwise from Table 8) to obtain last year's timing differences, and then multiplying those timing differences by this year's tax rate (from Table 8). Income taxes in Table 5 are computed by applying this year's tax rate from Table 8 to earnings before taxes (i. e., not including revaluation of deferred income taxes).

The statement of retained earnings is completed by invoking the ratio [book value target for financial strength] that was formulated in the previous section: The sum of deferred income taxes, common stock, and retained earnings should be 55.8% (rounded) of invested capital. However, negative dividends are not allowed (and by assumption a new issue of equity has also been ruled out). This means that ending retained earnings are set as the minimum of the following two:

$$\begin{aligned} & (\text{Beginning retained earnings}) + (\text{net income}), \\ & 0.558 \times (\text{invested capital}) - (\text{deferred income taxes}) - (\text{common stock}). \end{aligned}$$

Consequently dividends are the residual item in a forecasted statement of retained earnings:

$$\begin{aligned} \text{Dividends} &= (\text{beginning retained earnings}) + (\text{net income}) \\ &\quad - (\text{ending retained earnings}). \end{aligned}$$

The items in the forecasted balance sheets, Table 6, are to a large extent directly driven by revenues. That is, they are given by revenues multiplied by the relevant forecast assumptions in Table 8. This holds for the majority of the current assets items and for other current liabilities. Inventories and accounts payable are driven by operating expenses (and excess marketable securities are directly set to be zero; cf. Section 9).

Net PPE is also driven by revenues. Accumulated depreciation is last year's accumulated depreciation *plus* this year's depreciation (from Table 5) *minus* this year's retirements. This year's retirements equal (last year's net PPE) \times [retirements/last year's net PPE] (from Table 8). Gross PPE is then calculated as net PPE *plus* accumulated depreciation. It is again pointed out that gross PPE and accumulated depreciation are actually not needed. That is, rows 46 and 47 in Table 6 could have been left blank.

Short-term debt is specified as a fraction (from Table 8) of last year's long-term debt. Deferred income taxes are specified as (this year's net PPE) \times [timing differences/this year's net PPE] \times [this year's tax rate], as already mentioned in Section 8 above. Common stock is set to be the same as in year 0, as already explained. Retained earnings are copied from the same item in the statement of retained earnings in Table 5. Long-term debt then becomes the residual item, to obtain equality between assets and liabilities in each forecasted balance sheet. It is seen at the bottom of Table 6 that the 44.2% target for debt to invested capital that is implied by [book value target for financial strength] is reached in year 3, and that the ratio [NOPLAT/invested capital] is expected to be somewhat better on average than in recent historical years. All items in the forecasted income statements and balance sheets should be interpreted as expected values under some scenario.¹⁹

Finally, forecasted free cash flow for each year 1 to 12 is displayed in Table 7. That table is derived from Tables 5 and 6 in essentially the same fashion as Table 3 is derived from Tables 1 and 2. However, a comment on the role of the item revaluation of deferred income taxes is called for. Apparently, that item is included both in the income statements and in the free cash flow calculations. Suppose that the tax rate changes from 39% to 42% in year 4. If revaluation of deferred income taxes is not included in the free cash flow calculation, the free cash flow for year 4 in cell L107 becomes 8.8. (The reader can verify this by changing cell L91 from =K63-(K63/K170)*L170 to 0.) However, this is not the

¹⁹On the meaning of expected values under some scenario, see Jennergren 2013b. Briefly speaking, the forecasted income statements and balance sheets can be thought of as reverse engineered from a more complex valuation model that explicitly includes uncertainty, for instance, bankruptcy risk.

correct free cash flow. In particular, the free cash flow does not agree with the financial cash flow of 5.9. Actual cash taxes on the year's EBIT should be equal to taxes on EBIT in cell L89 minus the year's increase in timing differences multiplied by the tax rate that is valid for year 4. This product is equal to the change in deferred income taxes, but *only if the tax rate is the same in this year as in the previous year*. Hence, the correction term revaluation of deferred income taxes is necessary. In other words, the sum of revaluation of deferred income taxes (cell L91) and change in deferred income taxes (cell L93) is equal to the increase in timing differences multiplied by this year's tax rate.

The item revaluation of deferred income taxes in the income statement plays a different role. Again, suppose that the tax rate changes from 39% to 42% in year 4. If revaluation of deferred income taxes in cell L15 is deleted from the income statement (by changing cell L15 from $=K63-(K63/K170)*L170$ to 0), then that does not change the free cash flow (still equal to 5.9), but the financial cash flow now becomes 8.8. In particular, the common dividends increase from 9.2 to 12.2. What happens is a *violation of the clean surplus relationship*. Dividends are the residual in the statement of retained earnings (that is what the clean surplus relationship means in this case). The violation comes about because by deleting the item revaluation of deferred income taxes in cell L15, deferred income taxes are written up directly against retained earnings (i. e., against owners' equity). However, that write-up is neglected in the statement of retained earnings, since that statement assumes that the clean surplus relationship holds.

By depreciating PPE for tax purposes over a time period shorter than the economic life, a company can decrease its effective tax rate below the nominal rate, as long as nominal revenues are increasing. At the bottom of Table 7, the effective rate of taxes paid on EBIT is exhibited. That rate is computed by dividing (taxes on EBIT) *plus* (revaluation of deferred income taxes) *minus* (change in deferred income taxes) by EBIT. In steady state, the effective tax rate is apparently 36.2%, i. e., not much lower than the nominal rate of 39%.

11 Valuation of McKay's equity by the discounted cash flow model

Having forecasted the free cash flow for each year of the explicit forecast period and the first year of the post-horizon period, it is now possible to calculate the value of McKay's equity by means of the discounted cash flow model, in rows 194 - 216. As is clear from the heading to this segment of the value calculations part of MCK.XLS, the discounting is done using a WACC with constant weights, as will be explained later in this section of the tutorial. Two items that are necessary for the valuation are copied down, the book value of interest-bearing debt (short-term and long-term) at the beginning of each year (equal

to the end of the previous year; row 198), and the free cash flow (row 199). The latter is assumed to occur at the end of each forecast year. As already mentioned in Section 9, the market value of the interest-bearing debt is assumed equal to the book value.

The general procedure is the following: To begin with, the value of the firm's operations is computed as of the beginning of the first year of the post-horizon period, i. e., at the horizon. This value is obtained by the Gordon formula.²⁰ The free cash flow at the end of the first year in the post-horizon period (28.4) increases by a specified growth rate year by year over an infinite number of years. (The specified growth rate in the McKay example is 3%, due to inflation only, as already indicated earlier.) The WACC in the first year of the post-horizon period turns out to be 10.05% (rounded), so the result of the Gordon formula is $28.4/(0.1005-0.03) = 403.0$. How the WACC has been calculated will be discussed in greater detail below. The value of the operations 403.0 is the total value of the firm's assets (since there are zero excess marketable securities). From that total asset value is deducted the value of interest-bearing debt (201.5). The resulting equity value (including deferred income taxes) is 201.5. The debt and equity values are apparently equal. This is no coincidence, since a *target capital structure in market value terms* of 50% equity and 50% interest-bearing debt has been assumed, as will be seen shortly (indeed, this assumption was stated already in Section 9 above).

After that, a similar calculation is performed for the immediately preceding year, i. e., the last year of the explicit forecast period. The value 403.0 of the operating assets at the beginning of the following year, which is also the end of the current year, *plus* the free cash flow at the end of the current year (31.7) are discounted to the beginning of the current year, using the current year's WACC. Again, this provides the value of the firm's operations (395.0) at the beginning of the current year. Subtracting the debt value (197.6), one obtains the equity value at the beginning of the current year (197.5). The computations proceed in this manner, by discounting backwards year by year, until one reaches the beginning of the first year of the explicit forecast period which is also that moment in time when the valuation is done. Jumping to the conclusion, it is seen that McKay's equity (again including deferred income taxes) is valued in cell I204 at 98.2 as of Jan. 1 year 1.²¹

The computations apparently proceed backwards one year at a time. The value of the

²⁰Applying the Gordon formula to value the operations at the outset of the post-horizon period apparently assumes (among other things) an even age distribution of successive PPE cohorts. As mentioned in Section 7, that assumption cannot be expected to hold exactly. This inconsistency is usually not important. However, it is possible to set up a more complex infinite discounting formula that accounts for the precise timing of capital expenditures, cf. Jennergren 2008.

²¹Cell I208 contains the cum-dividend value of the equity, i. e., the equity value at the beginning of year 1 plus dividends at the end of year 0 (these two points in time are obviously the same). The cum-dividend value will be referred to in Section 15.

firm's operations at the beginning of any one year in the explicit forecast period is the present value of the sum of the value of the operations at the beginning of the following year *plus* this year's free cash flow. It is not difficult to see that this way of stepping backwards one year at a time gives the same result as directly discounting all yearly free cash flow amounts to a present value as of Jan. 1 year 1. However, the procedure suggested here is more general, since it permits the computation of equity value at the beginning of each year in the explicit forecast period, not only at the beginning of year 1.

The specification of the WACC is the standard one, well known from corporate finance texts. It is again convenient to introduce some notation:

- E market value of equity,
- D market value of debt,
- r_E nominal required rate of return on equity,
- r_D nominal cost of debt, assumed equal to the nominal borrowing rate,
- τ tax rate.

The WACC formula is then²²

$$r_E \frac{E}{D + E} + r_D (1 - \tau) \frac{D}{D + E}. \quad (8)$$

Equation (8) is the WACC formula that is used for the first year of the post-horizon period, year 12. The parameters r_D , r_E , and τ are given for each year in the forecast assumptions, Table 8. $E/(D + E)$ and $D/(D + E)$ are market value weights.²³ D is, by assumption, equal to the book value of the interest-bearing debt. The market value weights $E/(D + E)$ and $D/(D + E)$ should be valid for the *beginning* of that year to which the WACC formula is applied.

It is now possible to be more precise about the discounting operation in each year of the value calculation. For the first post-horizon period year, a *desired market value weight*

²²It is seen that the WACC is obtained in this tutorial as a weighted average of the *nominal* required rate of return on equity and the *nominal* after-tax cost of debt. An alternative procedure would be to take a weighted average of the *real* required rate of return on equity and the *real* after-tax cost of debt, and then adjust for expected inflation. The procedure in this tutorial actually follows from the assumption, mentioned in a footnote in Section 9, that the *before-tax* real borrowing rate remains constant under varying inflation expectations. The resulting value of WACC is somewhat lower than under the alternative procedure. Cf. again Howe 1992.

²³A comment on the terminology is called for here. In connection with valuation, "market value" is often used as a sloppy abbreviation for "computed market value" or "value as calculated by the valuation model". The idea is that a valuation model is used to calculate what the market value of the equity would be under some scenario. The equity value E that enters into the WACC formula (8) should hence be understood as the computed equity value according to the model, not the stock market value of the equity (if such a value can be observed).

of equity $E/(D + E)$ is specified in cell I210. The corresponding market value weight of debt is hence $D/(D + E) = 1 - E/(D + E)$. Apparently, it has been specified in this case that the *target capital structure in market value terms* should be 50% equity and 50% debt. Using those weights for debt and equity, the WACC is calculated in cell T200. It turns out to be 10.05%. With that WACC value, the value of the operating assets is determined (cell T202), as indicated earlier. Next, interest-bearing debt is subtracted, meaning that the equity value is obtained as a residual. At this point, the *resulting* market value weight of equity is determined in cell T212. (In other words, the contents of cell T212 is $=T204/(T198+T204)$.) Cell I211 contains a copy of cell T212.

The simultaneity problem that was mentioned in Section 2 above is now resolved, if the resulting $E/(D + E)$ in cell I211 is the same as the desired $E/(D + E)$ in cell I210. Cell I213 contains the difference between cells I210 and I211 multiplied by 1,000,000. The contents in cell I213 can be driven to zero, through a suitable choice, more precisely 55.8% (rounded), of the the ratio [book value target for financial strength] for year 1 in cell I183 in Table 8. If that target is changed for year 1, it also changes for years 2 through 12, since it is the same for all years in Table 8. Driving cell I213 to zero by adjusting cell I183 is most easily done using the Goal Seek procedure.

Equality between cells I210 and I211 implies a solution to the simultaneity problem. Resolving that problem actually does not affect the WACC for year 12, since that discount rate is, in any case, already determined by the desired weight $E/(D + E)$ that is specified in cell I210. Resolving the simultaneity problem only means adjusting the liabilities side of the balance sheet for year 12, so that the book value of interest-bearing debt becomes equal to its computed market value (being 50% of the market value of the company). At the same time the balance sheets for all previous years are also adjusted, since [book value target for financial strength] changes for all years in the explicit forecast period as well.

The WACC for each year in the explicit forecast period is calculated according to formula (8) as well, however *using the desired capital structure in market value terms with weights $E/(E + D)$ and $D/(E + D)$ as specified for the first year of the post-horizon period*. This is in line with a recommendation by Koller et al. (2010, p. 262): The estimated WACC should be founded on a target capital structure for the firm.²⁴ In this tutorial, that target capital structure is supposed to be attained at the outset of the post-horizon period, when the company develops in a steady-state fashion.

To summarize, the target capital structure is set for the first year of the post-horizon

²⁴The target capital structure considered here is in terms of market values. It is not the same as the ratio [book value target for financial strength] (interest-bearing debt should be 44.2% of invested capital) that was introduced in Section 9. However, the former is obviously related to the latter, since the latter is varied in the Goal Seek procedure, so that the former is attained in the first year of the post-horizon period. It may be noted that both the ratio [book value target for financial strength] and the target capital structure in market value terms are satisfied in every year of the post-horizon period.

period. The same capital structure is then imposed for the WACC in all preceding years (i. e., all years in the explicit forecast period), and also for the WACC in all subsequent years in the post-horizon period (through the Gordon formula discounting).

It is not excluded that the WACC can vary over the years in the explicit forecast period, even though each year's WACC uses the capital structure value weights $E/(E+D)$ and $D/(E+D)$ from the first year in the post-horizon period. The reason is, the other variables that enter into the WACC calculation can vary over individual years. Indeed, the relevant interest rate items as well as the tax rate are specified for each year separately in the forecast assumptions in Table 8.

With the model implementation suggested here, it is actually not even necessary to resolve the simultaneity problem that was mentioned above. That is, the capital structure in market value terms that is defined by the desired weight $E/(D+E)$ for the first year of the post-horizon period is sufficient to specify the WACC for every single year in the explicit forecast and post-horizon periods (given the other assumptions, i. e., borrowing rate, cost of equity capital, and tax rate). Free cash flow does not depend on the capital structure, as has already been mentioned in Section 9. Hence, the actual breakdown into debt and equity of the liabilities sides of the forecasted balance sheets does not matter. The breakdown into debt and equity at the valuation date does matter (since equity value is calculated as a residual), but that breakdown is taken from the last historical balance sheet, not from some forecast.

In the file MCK.XLS, there are apparently two applications of the Goal Seek procedure, to find the interpolation curve parameter that is related to [this year's net PPE/revenues] in the intermediate explicit forecast period years, and for setting [book value target for financial strength] to reach the desired capital structure in market value terms of 50% equity in the first post-horizon period year. A macro has been recorded that executes both Goal Seeks. This macro is called by pressing Control + Shift + G.

12 Sensitivity analysis: Valuation under different scenarios

The value of McKay's equity, found to be 98.2 in the previous section, is valid under that particular base case scenario that is defined by the forecast assumptions in Table 8 and the further assumptions (noted in Section 9) that were directly incorporated into the forecasted balance sheets.²⁵ Valuation results for some alternative scenarios are given

²⁵Koller et al. (2010, pp. 104, 650) recommend an adjustment to account for the fact that free cash flow on average occurs in the middle of each year, not at the end. This is accomplished by compounding one half year forward the computed value of the operating assets at the date of valuation. The compounding rate should be the first year's WACC. In the McKay case, that would result in a calculated value of the

Table A. McKay valuations under different scenarios

No.	Description of scenario	(a)	(b)	(c)
1	Base case	98.2	98.5	98.3
2	+ 1% real growth from year 10	98.8	99.2	98.9
3	+ 1% inflation from year 5	91.8	93.5	92.4
4	- 1% [operating expenses/revenues] from year 1	173.3	168.8	171.6
5	No improvement in working capital assets	93.8	94.6	94.0
6	Capital intensity parameter K 0.560 rather than 0.580	115.2	115.0	115.1
7	Economic life n of PPE 11 rather than 10 years	144.6	140.4	142.9
8	Tax rate 42% rather than 39% from year 1	86.6	88.4	87.3
9	+ 1% interest rates (borrowing and equity) from year 1	70.1	74.6	71.9

Explanations:

- (a) Discounted cash flow model
- (b) Discounted dividends model
- (c) Adjusted present value model

Table B. Cash flow from new investment

Cash flow element	10	11	12	13	14	15	16	17	18	19	20
Investment in PPE	-58.0										
Investment in working capital	-6.7	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	8.5
Revenues	100.0	103.0	106.1	109.3	112.6	115.9	119.4	123.0	126.7	130.5	
Operating expenses	-90.0	-92.7	-95.5	-98.3	-101.3	-104.3	-107.5	-110.7	-114.0	-117.4	
Taxes on revenues minus expenses	-3.9	-4.0	-4.1	-4.3	-4.4	-4.5	-4.7	-4.8	-4.9	-5.1	
Tax savings on fiscal depreciation		4.5	4.5	4.5	4.5	4.5					
Cash flow from investment	-58.6	10.7	10.8	11.0	11.2	11.4	7.1	7.3	7.5	7.7	8.5

in Table A. Column (a) shows results for the discounted cash flow model that has been presented so far in this tutorial. Columns (b) and (c) refer to the discounted dividends model and the adjusted present value model that are discussed in the following section.

Scenario 2 calls for a 1% increase in the real growth rate starting year 10, as compared to the base case scenario with zero real growth from that year. In other words, there is a growth opportunity under Scenario 2. It is seen that this growth opportunity has almost no value, since the computed equity value at the beginning of the first year goes up only slightly, from 98.2 to 98.8. (Incidentally, it is unrealistic to imagine that real growth of revenues in the post-horizon period could be higher than the expected long-term real growth of the surrounding economy as a whole.)

The conclusion that growth has almost no value can be checked by direct calculation, as in Table B. That table considers a growth project, an investment in year 10 that is supposed to support revenues of 100 (arbitrarily scaled) monetary units starting that year.

operating assets of 213.7×1.05025 , equal to 224.4. Deducting the value of interest-bearing debt 115.5, one obtains the computed equity value 108.9 rather than 98.2. This recommendation is not followed here, since the meaning of such an adjustment seems less clear for some of the other valuation models that are considered in the following three sections.

The necessary investment in PPE is then 58.0. Revenues increase by 3% per year, due to assumed inflation. The revenue stream goes on for 10 years, since that is the assumed economic life of the PPE. Operating expenses (being cash costs) are 90% of revenues. There is also investment in working capital, since the working capital items are driven by, i. e., proportional to, nominal revenues and operating expenses, as already explained in Section 4 above.²⁶ The investment in working capital is recouped at the end of the project. The tax consequences of the project are also listed in Table B. The last line of that table contains cash flow from the project starting year 10 and ending year 20. It may be noted that the amounts in Table B pertain to year ends and have been rounded to one decimal point.

If one discounts the cash flow in Table B to a present value at the beginning of year 10, at the WACC that is valid for year 10 and later years under the base case scenario, 10.05% (somewhat rounded), then the net present value is positive but very small (0.5). In other words, a growth opportunity has almost no value, for instance because the forecasted ratio [operating expenses/revenues] is rather high. This conclusion can also be reached by comparing the internal rate of return of the growth project in Table B to the WACC. The internal rate of return is 10.27%, which is close to the WACC 10.05%.²⁷

Scenario 3 considers a 1% increase in expected inflation, from 3% to 4%, from year 5. This leads to higher nominal cash flow but also to a higher WACC (since inflation enters into the nominal borrowing rate and the nominal cost of equity). However, higher nominal cash flow is not completely cancelled by a higher discount rate: Higher inflation increases the requirement for working capital. Also, tax savings due to depreciation of PPE do not increase in line with inflation, since they are based on the original acquisition prices. It is seen that the effect of a modest increase in expected inflation is negative, but rather small. This is not the end of the inflation story, however, since the increase in the nominal borrowing rate leads to a decrease in the value of already existing *fixed-interest* debt. In the base case scenario, all of the interest-paying debt has a nominal borrowing rate of 8.77%, and this is the going market rate for a company in McKay's risk class (that is why the book value of the debt could be deducted to arrive at the value of the equity as a residual). Suppose all of McKay's debt is fixed-interest. If so, the value of the debt falls, as inflation increases from 3% to 4%, due to the increase in the nominal borrowing

²⁶The necessary working capital can be calculated as 6.75% of revenues in year 10 and 6.70% of revenues in the following years. This implies the initial investment in working capital in year 10 and the additional investments in working capital in the following years until year 19 that are shown in Table B. The additional investments are driven by the 3% nominal growth in revenues that is due to inflation only. The figure 6.75% can be calculated from the cells in the file as $(R35+R38+R39+R40+R41-R56-R57)/R5$, and the figure 6.70% as $(S35+S38+S39+S40+S41-S56-S57)/S5$.

²⁷In connection with firm valuation, the internal rate of return of a growth project is sometimes referred to as the cash flow rate of return (Koller et al. 2010, pp. 180-183).

rate. The value of the equity, being the residual, goes up. Consequently, an increase in inflation implies a value transfer from interest-bearing debt to equity. No such value transfer has been considered in Scenario 3. For the result of Scenario 3 to be valid, one must hence assume that all of McKay's debt is *floating-rate*.

Scenario 4 shows that a 1% change in [operating expenses/revenues] has a very large impact on equity value. In fact, this ratio would seem to be the most critical forecast item in Table 8. Scenario 5 presumes that there is no improvement in the utilization of working capital assets over the explicit forecast period (remember that the base case assumed a 3% decrease in the working capital asset ratios). This has only a small effect on the computed equity value.

The following two scenarios consider effects of revised assumptions as regards PPE productivity. In Scenario 6, it is assumed that there is an improvement (i. e., decrease) in the capital intensity parameter K over the explicit forecast period. That is, K falls to 0.560. This affects [this year's net PPE/revenues] in year 11 and also in earlier years, because of interpolation. The revision of K in this scenario from 0.580 to 0.560 has a fairly important effect on the equity value, considering that it is really quite small (-0.020). In Scenario 7, the economic life of the PPE at the horizon is set to 11 rather than 10 years (it is remembered from Section 7 that n was estimated from the company's historical financial statements as either 10 or 11, but 10 was chosen for the base case). A change in assumed economic life of PPE induces changes in [this year's net PPE/revenues] and also in [depreciation/last year's net PPE] and [timing differences/this year's net PPE]. The resulting impact on equity value is again seen to be quite important.

A moderate tax rate increase (Scenario 8) has only a moderate effect on computed equity value. Free cash flow is reduced, as the tax rate is increased, but there is a counter-acting force through deferred income taxes. A tax rate change also finds its way into the WACC calculation, through formula (8) in the previous section.

Scenario 9 emphasizes the importance of a single percentage point in the discount rate (WACC) in connection with firm valuation. A 1% increase in the discount rate without an accompanying increase in expected inflation could come about through similar increases in the real borrowing rate and the real cost of equity capital. The previous caveat about all the debt being floating-rate applies here, as well.

13 The discounted dividends model and the adjusted present value model

Two more valuation models are illustrated in the file MCK.XLS. The second segment in the value calculations part (rows 218 - 224) is concerned with the *discounted dividends model*, with a cost of equity that is consistent with constant WACC weights. This is a

very straight-forward valuation model: Dividends, taken from row 27 (in the statement of retained earnings) are copied in row 221, and the cost of equity, taken from row 179 in Table 8, is copied in row 222. The reader is reminded that this cost of equity is under the assumption that the equity weight in the WACC (i. e., the capital structure in market value terms) is constant, in fact, 50% (as discussed in Sections 9 and 11 above). This does not exclude that the cost of equity can change for some other reason than a change in the capital structure, e. g., a change in expected inflation. Dividends are simply discounted backwards year by year, starting with the Gordon formula for the post-horizon period. The value of the equity at the start of year 1 under the base case scenario apparently is 98.5. Column (b) in Table A reports calculated equity values according to the discounted dividends model under all of the investigated scenarios.

The next model is the *adjusted present value model* (rows 226 - 242) (cf. Berk and DeMarzo 2011, pp. 601-604; Brealey et al. 2011, pp. 514-518; Ross et al. 2008, pp. 488-500; Ruback 2002). Row 231 shows each year's *tax shield*, being the tax saving that the company obtains by deducting the yearly debt interest. The value of the operating assets is calculated in two steps. First, an unlevered value of the operations is calculated, by discounting free cash flow at the *opportunity cost of capital*, which is the same as the cost of equity capital under the assumption of equity financing only. Hence, the value of the operating assets is calculated as if these assets were financed only by equity. In the second step, the tax shields from partial debt financing in row 231 are also discounted to a present value using the opportunity cost of capital as discount rate. In other words, the present value of the tax shields is viewed as a side effect that is added to the value of the operating assets if financed only by equity.

The opportunity cost of capital is denoted by r_U and is computed year by year in row 232 as

$$r_U = r_E \frac{E}{D + E} + r_D \frac{D}{D + E} = r_E \times 0.5 + r_D \times 0.5. \quad (9)$$

As in equation (8) in Section 11 above, $E/(D + E)$ and $D/(D + E)$ are market value weights. r_E and r_D are given in the forecast assumptions, rows 179 and 177 of Table 8. Note the difference compared to (8): The debt term in (9) is not multiplied by $(1 - \tau)$. The right hand side of the second equality in (9) shows that the market value weights that are associated with r_E and r_D from Table 8 are 50% equity and 50% debt, as already mentioned earlier. Using the opportunity cost of capital rather than the borrowing rate for tax shield discounting is appropriate here, since the debt tax shields should be considered as uncertain (cf. the earlier statement in Section 10 that all items in the forecasted financial statements should be interpreted as expected values under some scenario).

Row 240 adds the computed unlevered value and the computed tax shield value at the start of each year to provide the value of the operating assets including tax shields.

Deducting the debt, one gets the equity value as a residual, like in the discounted cash flow model. The computed equity value at the start of year 1 is seen to be 98.3. Column (c) in Table A shows equity values computed by the adjusted present value model for all of the different scenarios.

Comparing the three models for which results are reported in Table A, it is clear that they do not provide exactly the same equity values. As can be checked in the file MCK.XLS, however, they provide exactly the same values at the outset of the post-horizon period. The slight differences that are seen in Table A are hence due to inconsistencies in the treatment of the explicit forecast period. It should be noted that the financing policy is the same for all three valuation models. In particular, the ratio [book value target for financial strength] 55.8% is the same for all three models, meaning that the absolute amounts of debt are the same, year by year, in all three models (necessarily so, since only one set of forecasted balance sheets has been generated). The differences in computed results must hence be due to inconsistencies in the discounting.

14 Valuation with variable WACC weights to resolve the simultaneity problem in every year

The file MCK.XLS contains two more valuations, in the fourth and fifth segments of the value calculations part. In the fourth segment in rows 244 - 260, the company is revalued by the discounted cash flow model with a WACC where the weights are varying in all years of the explicit forecast period, rather than being fixed at 50% each of equity and debt, as before. More precisely, the weights are set in each year in such a fashion that the weights that result from the valuation agree with the weights that are used in the WACC to derive that valuation. This means that the simultaneity problem that was discussed in Section 11 is resolved in every year of the explicit forecast period.

The WACC calculation necessarily becomes more complex. Since the WACC weights are varying over the explicit forecast period years, one needs to specify how the borrowing rate and the required rate of return on the equity vary with the capital structure. For the borrowing rate, it is simply assumed that it remains constant as in row 177 and hence does not depend on the capital structure. This may be a reasonable assumption for small variations in the WACC weights. For large variations, one would want to write the borrowing rate as a function of the WACC weights (however, to define a reasonable such function may not be evident).

For the cost of equity, the following relationship is used:

$$r_E = r_U + (r_U - r_D) \frac{D}{E}. \quad (10)$$

Here, the opportunity cost of capital r_U is given by (9) in the previous section. The debt to equity ratio D/E should be valid for the beginning of that year to which (10) is applied. The equation (10) agrees with the famous “MM’s Proposition 2” (Berk and DeMarzo 2011, pp. 460-466; Brealey et al. 2011, pp. 452-456; Ross et al. 2008, pp. 430-439) and is also consistent with the adjusted present value model where expected tax shields from partial debt financing are discounted at the opportunity cost of capital (see Ruback 2002; cf. also Taggart 1991).

The discounting proceeds as follows: Setting [book value target for financial strength] to 55.8% in the fashion explained earlier (Section 11) insures that one reaches the desired capital structure in market value terms for the first post-horizon period year of 50% equity and 50% debt. The step-wise discounting backwards one year at a time in the explicit forecast period calls for the following calculation in each year t :

$$E_t + D_t = \frac{f_t + D_{t+1} + E_{t+1}}{1 + \left(r_U + (r_U - r_D) \frac{D_t}{E_t} \right) \frac{E_t}{D_t + E_t} + (1 - \tau) r_D \times \frac{D_t}{D_t + E_t}}. \quad (11)$$

E_t denotes the computed equity value at the start of year t . D_t is the market value, equal to book value, of the debt at the start of the same year. Hence, $E_t + D_t$ is the computed value of the operating assets at the start of year t (equal to the sum of equity plus debt). f_t is the free cash flow at the end of year t . The value of the operations at the start of year t is apparently equal to the sum of free cash flow at the end of the current year plus the computed value of the operating assets at the start of the following year, discounted back one year, with the WACC that is seen in the denominator as discount rate. It is clear that the cost of equity in the WACC is given by (10).

The simultaneity problem is visible in (11): The object is to determine $E_t + D_t$ on the left hand side, but E_t also enters into the right hand side. However, one can solve for E_t and move D_t back to the left hand side to obtain

$$E_t + D_t = \frac{f_t + D_{t+1} + E_{t+1} + \tau r_D D_t}{1 + r_U}, \quad (12)$$

so (12) is hence the value of the operations at the start of year t . This is precisely the contents of the cells in row 253 for the explicit forecast period years. Deducting the debt, one obtains the value of the equity as a residual in row 255, as usual.

The computed equity value at the start of year 1 according to the discounted cash flow model with non-constant WACC weights is 98.3, exactly equal to the equity value according to the adjusted present value model. In fact, this equality holds for every year. As will be argued shortly, this is no coincidence. The resulting equity weights and the resulting WACC are shown in rows 258 and 259. The resulting cost of equity is shown in row 260 and is used in the discounted dividends model that is consistent with non-constant WACC weights.

Consider therefore, finally, the discounted dividends model where the required rate of return on the equity varies in each explicit forecast period year in accordance with (10). This variant of the discounted dividends model is contained in the fifth segment (rows 262 - 268) of the value calculations part of MCK.XLS. Discounting expected dividends at the cost of equity in row 266 that is equal to row 260 for the explicit forecast period years, again one finds the computed value of the equity to be 98.3, exactly the same as according to the adjusted present value model.

Hence, the adjusted present value model, the discounted cash flow model with a non-constant WACC, and the discounted dividends model with a cost of equity that is consistent with a non-constant WACC give identical computed equity values. This can be checked in all of the scenarios that are listed in Table A. It is well known that a variety of valuation models provide identical results, given that one is consistent (cf. Levin 1998, Young et al. 1999). Consistency does not hold in the first three segments of the value calculations part of MCK.XLS, since there the WACC weights in the explicit forecast period years are fixed in advance to 50% equity and 50% debt, regardless of whether that is a solution to the simultaneity problem in every explicit forecast period year. Consistency does hold in the last three segments of the value calculations part of MCK.XLS, i. e., with variable WACC weights that solve the simultaneity problem in every year.

In particular, as mentioned earlier, the adjusted present value model with debt interest tax shields discounted at the opportunity cost of capital is consistent with the discounted dividends model, if the required rate of return on the equity is specified as in (10). The adjusted present value model is also consistent with the discounted free cash flow model, if the required rate of return on the equity entering into the WACC is set as in (10), and if the WACC weights resolve the simultaneity problem (indeed, this can be seen in (12)). Similarly to the remark at the end of the previous section, it is noted here that it is only the discounting that varies between the different models and model variants in all five segments of the value calculations part of the file MCK.XLS. The forecasted financial statements underlying the calculations are exactly the same in all five cases. In particular, there is no difference in forecasted debt levels between the discounted cash flow model with a constant WACC and the same model with a non-constant WACC. The difference between these two valuations has to do only with the manner in which the WACC is determined.

One may now pose the question: Which variant of the discounted cash flow model is preferable, the one with constant WACC weights in all explicit forecast period years, or the one with non-constant WACC weights that solve the simultaneity problem in every explicit forecast period year? A first observation is that the difference in resulting equity value between the two alternatives is very small in the case of McKay, so the choice is actually not very important. The inconsistency that follows from constant WACC weights

may seem annoying (see row 212 that shows that the resulting WACC weights are not exactly equal to 50% in the explicit forecast period years, in particular year 1). For simplicity, one may nevertheless stick to constant WACC weights.

There is a fairly important weakness associated with varying the WACC weights as in the fourth segment of the value calculations part of the file: As will be remembered from the discussion in Section 9, the ratio [book value target for financial strength] was motivated by McKay's rather weak performance in recent historical years. In particular, the debt level had been observed to increase quite markedly. The company should try to gain financial strength. Indeed, this happens in the first two years of the base case scenario, since no dividends are paid out in those years. However, as the financial strength improves, the resulting WACC with non-constant weights in row 259 *increases* (although only slightly, it must be admitted). This seems counterintuitive and may not encourage the management of the company to strive to improve the financial strength in the first place. The problem is that we lack a comprehensive theory for how the cost of capital should vary with the capital structure, taking into account aspects such as higher borrowing costs when financial strength falls, and agency and bankruptcy costs.

15 A new financing policy, and two further valuation models

The discussion in this section refers to the file MCK_EXT.XLS. Compared to the McKay valuation earlier in this tutorial, there are two new features (otherwise there are no changes from the file MCK.XLS).

In the first place, the financing policy that is adopted now prescribes that the equity weight in the capital structure should be 50% in market value terms *in every single year*, including all years in the explicit forecast period. The means to accomplish this financing policy is to set the debt in every forecast year accordingly. Consequently, the ratio [book value target for financial strength] that is varied in MCK.XLS to attain the desired capital structure in market value terms at the beginning of the first post-horizon period year is deleted. Instead, each year's long-term debt in row 62 is set so that the resulting market value weight of equity is equal to the desired market value weight that is valid for that year (except in year 12; see below). The year by year desired market value weights for equity are specified in row 207. The resulting market value weights are computed in row 208. Row 209 contains the absolute deviations between rows 207 and 208. In cell I210, all of the elements in row 209 are summed, and the sum is multiplied by 1,000,000. The value of cell I210 is driven to zero through the Excel Solver, by adjusting each year's long-term debt in cells H62 through S62. Total common equity (row 69) then becomes the balance sheet residual. This means that dividends (row 27) or new share issue (row 28) are the residual

items in the statements of common equity. Since there is now the possibility of new share issues, the statements of retained earnings in Tables 1 and 5 of the file MCK.XLS have been expanded to statements of (total) common equity in MCK_EXT.XLS.

Apparently, this means that the long-term debt in the last historical year (in cell H62) is also adjusted, so that the capital structure in market value terms becomes 50% debt and 50% equity at the start of year 1 (which is equal to the end of the last historical year). Comparing the computed equity value at the start of year 1 in MCK_EXT.XLS (cell I201) and in the first segment of the value calculations part of MCK.XLS (cell I204), it is seen that the former is 106.9 and the latter 98.2. However, the long-term debt in MCK_EXT.XLS in the last historical year is smaller than that of MCK.XLS. In fact, comparing the dividends in the last historical year of MCK_EXT.XLS and MCK.XLS, it is seen that there is a dividend paid out in MCK.XLS of 2.9. In MCK_EXT.XLS, there is a new share issue of 5.7, shown in cell H28. The *cum-dividend equity values* at the start of year 1 in cell I205 of MCK_EXT.XLS and in cell I208 of MCK.XLS are *exactly equal*, 101.1. In a certain sense, the two financing policies in MCK.XLS and MCK_EXT.XLS are therefore equivalent.²⁸

When the long-term debt in the last historical year (in cell H62 of MCK_EXT.XLS) is set to attain the target capital structure in market value terms, total common equity in the last historical year (in cell H69) becomes the balance sheet residual. The common dividends or the new share issue in year 0 are then determined through the statement of common equity for that year. In other words, the computed value of the equity 106.9 at the beginning of year 1 in MCK_EXT.XLS pertains to a situation where the owners have already contributed equity capital to reach the desired capital structure in market value terms at the beginning of year 1.²⁹

The second new feature of MCK_EXT.XLS is that it includes valuations by two additional models, so there are valuations by five models in all. The first one out of the five is free cash flow and has already been commented on. The second one is *economic value added*. There are two variants of this model. In the first variant, which agrees with Koller et al. 2010, the economic value added in each year is NOPLAT minus a capital charge, given by WACC times the year's starting invested capital (Koller et al. 2010, pp. 101-102, 115-119; these authors use the designation economic profit). The value of the operating assets is then equal to the year's starting invested capital *plus* the present value (with WACC for discounting) of all (year by year) subsequent economic value added. Deducting the value of the interest-bearing debt, the equity value is obtained as a residual, like in the

²⁸The designation cum-dividend equity value is used by Penman (2010, p. 79), among others.

²⁹The long-term debt at the end of year 12 (the first year of the post-horizon period) is set by imposing the same book value ratio between interest-bearing debt and invested capital in year 12 as in year 11 (cell S75), since that ratio does not change in the post-horizon period.

free cash flow model. The second variant takes as a starting point EBIT minus taxes on EBIT and then deducts a capital charge that is equal to WACC times the year's starting invested capital minus deferred income taxes (in other words, deferred income taxes are considered to be a liability part of invested capital). As in the first variant, the value of the operating assets is equal to the year's starting invested capital (with deferred income taxes deducted) *plus* the present value of all (year by year) subsequent economic value added.

The third model is adjusted present value. The valuation is very similar to the valuation by the same model in the file MCK.XLS. It should be noted, however, that the tax shields in row 257 of MCK_EXT.XLS are not the same as the tax shields in row 231 of MCK.XLS. The reason is, of course, that the forecasted balance sheets, in particular the debt amounts, are not exactly the same in the two files.

The fourth model is discounted dividends and needs no comments. The fifth model is *abnormal earnings*. Abnormal earnings are net income minus a capital charge. The capital charge is the book equity value at the start of the year multiplied by the required rate of return on the equity. The computed equity value is the book equity value at the start of the year plus the computed value of all subsequent abnormal earnings (with discounting at the required rate of return on the equity). It may be noted that the computed value of subsequent abnormal earnings in row 285 is the same as the computed value of subsequent economic value added in the second variant of the economic value added model (row 244). This is a consequence of the fact that the equity owners are in a residual position. In the end, all of the economic surplus goes to the share holders, after paying the required debt service to the debt holders.³⁰

It is noted that *all five models give identical results*. In other words, the computed equity value is equal to 106.9 in all five cases. This is another example of valuation models providing identical results, given that one is consistent. A further remark on consistency can be made here: The five models in the file MCK_EXT.XLS can be divided into three groups, (i) the discounted cash flow and the economic value added models; (ii) the adjusted present value model; and (iii) the discounted dividends and the abnormal earnings models. It is known that the economic value added model is equivalent to the free cash flow model. Hence, these two models always give identical results. Also, the abnormal earnings model is equivalent to the discounted dividends model, so these two models always give identical results, too. However, equivalence between the three groups of models holds only if one is consistent. Consistency is satisfied in the present file MCK_EXT.XLS, since the desired capital structure in market value terms is imposed in every single year, in particular in every single year of the explicit forecast period. Consistency also holds in the last three segments of the value calculations part of MCK.XLS, since the WACC weights are

³⁰Rows 229, 230, 249, 250, 288, and 289 are references to a companion tutorial, Skogsvik 2002.

adjusted there year by year to ensure that the simultaneity problem is resolved in every year, as was mentioned in the previous section.

In MCK_EXT.XLS, there is apparently one application of the Goal Seek procedure to set the curve parameter α for the non-linear interpolation of [this year's net PPE/revenues] in intermediate explicit forecast period years, and one application of the Solver to reach the target capital structure in market value terms in every forecast year. A macro has been recorded that executes both applications. That macro can be called by pressing Ctrl + Shift + I. In order to enable the Solver to drive cell I210 to zero (or close to zero), it may be required to press this combination of keys several times. Actually, the Solver is not necessary in MCK_EXT.XLS. Instead, one could have specified the cost of equity as in equation (10) and then proceeded in an equation-solving manner, analogously to the previous section (in the discussion of the discounted cash flow model with non-constant WACC weights). Nevertheless, the Solver is used in this section as an interesting illustration of how it can be applied in valuation situations.

16 Concluding remarks

It is now clear that the discounted cash flow model cannot be viewed as a precise prescription of how to proceed when valuing a company. A similar remark also holds for other valuation models, like the economic value added model. On the contrary, a number of modelling choices must be made when implementing a valuation model. In conclusion, some of those choices will be commented on.

McKay's *excess marketable securities* were assumed to be sold off already during the last historical year, i. e., they were set to zero in all forecast years. This is a convenient assumption for the purpose of valuation, at least for a company with only a moderate portfolio of such securities, and even if there is no actual intention on the part of the company to dispose of that portfolio. The *financing policy* that was initially assumed in the McKay example, to use an adjustable ratio [book value target for financial strength] to attain a target capital structure in market value terms in the first year of the post-horizon period, is only one of several possible choices. Section 15 illustrated another possible financing policy.

It has been indicated above (Sections 5 - 8) how free cash flow in the post-horizon period can be forecasted in a consistent fashion, through particular settings of forecast ratios relating to PPE and deferred income taxes in the last year of the explicit forecast period. It is not so easy to specify what are *consistent forecast ratios in the earlier years of the explicit forecast period* (cf. Jennergren 2010). This tutorial has suggested the heuristic device of setting those ratios through interpolation starting from historical averages at the beginning of the explicit forecast period.

For *discounting in the post-horizon period*, this tutorial has used the Gordon formula. Koller et al. (2010, pp. 37-41, 212-215) favor a different approach, the so-called value driver formula. They admit that either formula is in a technical sense equivalent to the other. However, applying the Gordon formula (referred to as the free cash flow perpetuity formula by these authors) is tricky, leading to mistakes by many analysts. They say that “The typical error is to estimate incorrectly the level of free cash flow that is consistent with the growth rate being forecast” (pp. 213-214). The purpose in this tutorial of modelling the forecast ratios relating to PPE in the last year of the explicit forecast period as depending on four parameters g (real growth), i (inflation), n (PPE economic life), and K (capital intensity) has been to enable an application of the Gordon formula that is at least reasonably transparent and consistent. The approach in this tutorial is therefore suggested as an interesting alternative to the value driver formula that is proposed by Koller et al. Also, the value driver formula is not unproblematic in itself (see Jennergren 2013a).

Yet another possibility is to make explicit forecasts of income statements and balance sheets for a large number of individual years and then discount free cash flow for each year separately. Some reflection on this latter possibility leads to the conclusion, however, that there are actually no less than *three different horizon concepts*, or perhaps more clearly: horizon years, in firm valuation models. In the first place, there is the first year for which no forecast assumptions (like in Table 8) change from the previous year. In the second place, there is that year that defines the market value weights for equity and interest-bearing debt, i. e., for the capital structure in the WACC. In the third place, there is the last year for which an explicit income statement and balance sheet are forecasted (i. e., infinite discounting of one variety or another, or some other terminal value, is used beyond that year). In the McKay example, all of these three different horizon years are the same and equal to the first year of the post-horizon period (year 12). However, in more general set-ups, these three horizon years need not coincide.

Yet another choice relates to *what model should be used*. As seen in Sections 13 - 15, there now exists a variety of models that are similar in that they operate on forecasted income statements and balance sheets. What they discount, and at what discount rate, varies from one model to another. If one is consistent, they provide the same computed equity value. However, in an actual implementation that may be more or less heuristic, the various models may give results that do not exactly agree. In other words, the choice of a model may affect the final result.

So there are a number of modelling choices to be made. A fair amount of judgment on the part of the analyst doing the valuation is always required. Hence, firm valuation is more art than exact science. This is, in fact, a statement that has often been made in connection with this fascinating topic (see, e. g., Thomas and Gup 2010, pp. 411, 471;

Koller et al. 2010, p. 302; Jennergren and Näslund 1996, p. 57).

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MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

A	B	C	D	E	F	G	H	TABLE 5. FORECASTED INCOME STATEMENTS												S	T
TABLE 1. HISTORICAL INCOME STATEMENTS								I	J	K	L	M	N	O	P	Q	R				
								1	2	3	4	5	6	7	8	9	10				
1																					
2																					
3	Income statement	-6	-5	-4	-3	-1	0														
4	Revenues	197.6	222.3	272.3	299.5	418.9	505.4	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1229.4	1266.3	1304.3		
5	Operating expenses	-175.4	-205.8	-249.6	-274.7	-320.5	-467.4	-538.8	-621.5	-691.4	-762.0	-831.9	-899.7	-963.8	-1022.5	-1074.2	-1106.5	-1139.7	-1173.9		
6	Depreciation	-12.8	-9.3	-11.2	-13.0	-15.0	-26.4	-28.7	-32.0	-36.6	-40.2	-43.8	-47.2	-50.4	-54.3	-57.9	-61.2	-63.3	-64.5		
7	Operating income	9.4	7.2	11.5	11.8	14.5	17.6	31.1	37.1	40.3	44.4	48.6	52.7	56.7	59.3	61.5	61.8	63.3	65.9		
8	Interest income	0.0	0.0	0.0	0.0	0.9	0.7	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
9	Interest expense	-0.1	-0.4	-0.8	-1.0	-3.4	-4.1	-10.1	-10.2	-11.2	-12.1	-13.0	-13.9	-14.6	-15.5	-16.3	-17.0	-17.3	-17.7		
10	Earnings before taxes	9.3	6.8	10.7	10.8	12.0	14.2	21.0	26.9	29.1	32.3	35.6	38.8	42.1	43.8	45.1	44.8	46.0	48.2		
11	Revaluation of deferred income taxes							0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
12	Income taxes	-3.3	-2.4	-3.8	-4.2	-5.0	-6.1	-8.2	-10.5	-11.3	-12.6	-13.9	-15.1	-16.4	-17.1	-17.6	-17.5	-17.9	-18.8		
13	Net income	6.0	4.4	6.9	6.6	7.0	8.1	12.8	16.4	17.7	19.7	21.7	23.7	25.7	26.7	27.5	27.3	28.1	29.4		
14	Statement of retained earnings	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
15	Beginning retained earnings																				
16	Earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5	85.3	101.7	112.3	122.1	131.0	138.8	148.0	156.0	162.5	168.5	174.2		
17	Dividends																				
18	Ending retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5	85.3	101.7	112.3	122.1	131.0	138.8	148.0	156.0	162.5	168.5	174.2		
19																					
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MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
51																				
52	Total assets	102.8	116.4	149.2	159.9	200.3	269.8	284.6	317.4	359.3	392.0	423.6	453.3	480.4	510.7	537.8	560.8	573.2	585.9	603.5
53																				
54																				
55	Short-term debt	0.3	0.8	0.9	1.5	11.5	12.5	20.7	19.0	19.4	21.7	23.4	25.1	26.7	28.1	29.8	31.3	32.5	33.0	33.7
56	Accounts payable	7.3	11.0	11.9	10.5	14.2	16.2	18.9	23.7	27.3	30.4	33.5	36.6	39.6	42.4	45.0	47.3	48.7	50.1	51.6
57	Other current liabilities	13.9	13.5	18.2	18.5	21.4	27.8	28.8	38.0	43.8	48.7	53.7	58.7	63.4	68.0	72.1	75.7	78.0	80.4	82.8
58																				
59	Total current liabilities	21.5	25.3	31.0	30.5	47.1	56.5	68.4	80.6	90.6	100.8	110.6	120.3	129.7	138.4	146.9	154.3	159.2	163.5	168.1
60																				
61	Long-term debt	5.5	11.5	16.2	21.7	40.2	90.6	94.8	97.1	108.3	116.8	125.5	133.3	140.3	149.1	156.3	162.4	165.1	168.5	173.8
62																				
63	Deferred income taxes	8.7	11.0	12.6	15.5	19.6	25.1	25.3	30.8	35.1	38.5	41.9	45.1	48.0	51.6	55.0	58.0	59.9	61.9	63.7
64																				
65	Common stock *	4.6	4.9	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6	23.6
66	Retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	72.5	85.3	101.7	112.3	122.1	131.0	138.8	148.0	156.0	162.5	165.5	168.5	174.2
67																				
68	Total common equity	67.1	68.6	89.4	92.2	93.4	97.6	96.1	108.9	125.3	135.9	145.7	154.6	162.4	171.6	179.6	186.1	189.1	192.1	197.8
69																				
70	Total liabilities and equity	102.8	116.4	149.2	159.9	200.3	269.8	284.6	317.4	359.3	392.0	423.6	453.3	480.4	510.7	537.8	560.8	573.2	585.9	603.5
71																				
72																				
73	Invested capital	70.7	88.9	98.6	120.6	164.7	220.0	236.9	255.7	288.1	312.8	336.4	358.1	377.3	400.4	420.7	437.8	446.5	453.4	469.1
74	Debt/invested capital	-7.2%	10.5%	-3.4%	10.7%	31.4%	44.2%	48.8%	45.4%	44.3%	44.2%	44.2%	44.2%	44.2%	44.2%	44.2%	44.2%	44.2%	44.2%	44.2%
75	NOPLAT/invested capital																			
76	(ROIC)		8.7%	9.6%	9.2%	8.9%	8.1%	3.2%	9.9%	9.9%	9.3%	9.4%	9.5%	9.5%	9.8%	9.6%	9.4%	9.0%	9.0%	9.1%
77																				
78	* direct forecast																			
79																				
80																				
81																				
82	Free cash flow		-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
83																				
84	Revenues		222.3	272.3	299.5	350.0	418.9	505.4	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1229.4	1266.3	1304.3
85	Operating expenses		-205.8	-249.6	-274.7	-320.5	-383.6	-467.4	-538.8	-621.5	-691.4	-762.0	-831.9	-899.7	-963.8	-1022.5	-1074.2	-1106.5	-1139.7	-1173.9
86	Depreciation		-9.3	-11.2	-13.0	-15.0	-17.7	-26.4	-28.7	-32.0	-36.6	-40.2	-43.8	-47.2	-50.4	-54.3	-57.9	-61.2	-63.3	-64.5
87																				
88	EBIT		7.2	11.5	11.8	14.5	17.6	11.6	31.1	37.1	40.3	44.4	48.6	52.7	56.7	59.3	61.5	61.8	63.3	65.9
89	Taxes on EBIT		-2.6	-4.1	-4.6	-6.0	-7.4	-4.4	-12.1	-14.5	-15.7	-17.3	-19.0	-20.6	-22.1	-23.1	-24.0	-24.1	-24.7	-25.7
90	Revaluation of deferred																			
91	income taxes								0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
92	Change in deferred																			
93	income taxes		2.3	1.6	2.9	4.1	5.5	0.2	5.5	4.3	3.5	3.4	3.2	2.9	3.6	3.4	3.0	1.9	2.0	1.9
94																				
95	NOPLAT		6.9	9.0	10.1	12.6	15.7	7.4	24.4	26.9	28.0	30.5	32.8	35.1	38.2	39.6	40.5	39.6	40.6	42.1
96	Add back depreciation		9.3	11.2	13.0	15.0	17.7	26.4	28.7	32.0	36.6	40.2	43.8	47.2	50.4	54.3	57.9	61.2	63.3	64.5
97																				
98	Gross cash flow		16.2	20.2	23.1	27.6	33.4	33.8	53.2	58.9	64.6	70.7	76.7	82.3	88.6	93.8	98.4	100.8	103.9	106.6
99																				
100	Change in working																			

MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
101	capital (+/- if increase)		5.1	5.6	2.4	10.4	2.5	8.3	0.5	6.3	5.2	5.1	5.0	4.8	4.4	3.9	3.3	1.8	1.8	2.5
102	Capital expenditures																			
103	(+ if positive)		22.4	15.3	32.6	48.7	70.5	35.0	47.0	58.1	56.2	58.6	60.5	61.7	69.0	70.7	71.6	68.1	70.4	75.6
104	Gross investment								47.5	64.4	61.3	63.8	65.5	66.5	73.4	74.6	75.0	69.9	72.2	78.2
105	Free cash flow								5.7	-5.5	3.3	6.9	11.1	15.8	15.2	19.2	23.4	30.9	31.7	28.4
106	Financial cash flow																			
107	Incr(+)/Deer(-) in excess																			
108	marketable securities																			
109	After-tax interest																			
110	income (-)																			
111	Incr(+)/Deer(-) in short-																			
112	and long-term debt																			
113	After-tax interest																			
114	Common dividends (+)																			
115	Incr(+)/Deer(+)																			
116	in common stock																			
117	Financial cash flow																			
118	Effective tax rate on EBIT																			
119																				
120																				
121																				
122																				
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MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
151	Equipment (PPE)																			
152	PPE economic life (n)																			10
153	F _{sub g} (see Sect 6)																		10.0000	
154	F _{sub c} (see Sect 6)																		8.7861	
155	Real PPE/rev's (K)																		0.580	
156	M (see Section 6)																		0.510	
157	H (see Section 6)																		0.426	
158	Net PPE/revenues	31.5%	33.9%	29.2%	33.1%	37.9%	44.3%	38.4%	35.5%	34.5%	33.6%	32.7%	31.7%	30.8%	30.5%	30.2%	29.9%	29.6%	29.3%	29.3%
159	Depreciation/net PPE *	14.9%	14.9%	14.9%	16.4%	15.1%	13.3%	14.2%	14.8%	15.1%	15.3%	15.6%	15.8%	16.1%	16.4%	16.6%	16.9%	17.2%	17.4%	17.4%
160	Retirements/net PPE *	7.5%	6.4%	6.5%	-0.4%	2.0%	2.0%	5.3%	5.3%	6.3%	7.3%	8.3%	9.3%	10.3%	11.2%	12.2%	13.2%	14.2%	15.2%	15.2%
161	Net PPE/revenues curve parameter (α)																			
162	Inflation factors								1.344	1.305	1.267	1.230	1.194	1.159	1.126	1.093	1.061	1.030	1.000	
164	Residualing real gross PPE																		734.4	
165	Required real gross PPE								0.000										734.4	
166	Drive to 0 by Goal Seek (vary S162)!																			
168	Taxes																			
169	Tax rate	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%
170	PPE tax life (n)																		5	
171	PPE tax life (n)																		2.160	
172	J (see Section 8)																			
173	Timing diffrs/net PPE	35.8%	37.4%	40.6%	40.1%	37.8%	34.7%	33.4%	37.1%	37.7%	38.3%	38.8%	39.4%	40.0%	40.5%	41.1%	41.7%	42.2%	42.8%	42.8%
174	Interest rate items																			
175	Real borrowing rate								5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%
176	Nominal borrowing rate								8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%
177	Real cost of equity								11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%
178	Nominal cost of equity								14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%
179	Implied borrowing rate	6.90%	6.50%	5.85%	5.85%	14.66%	7.93%	9.80%												
181	Book value target for financial strength								55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%	55.8%
182	This year's short-term/last year's long-term debt								20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
183	* last year's net PPE																			
184																				
185																				
186																				
187																				
188																				
189																				
190																				
191																				
192																				
193																				
194	Discounted Cash Flow																			
195	constant WACC weights																			
196	Interest-bearing debt (at year start)																			
197	Free cash fl (at year end)																			
198	WACC								11.55	11.60	12.77	138.4	148.8	158.4	166.9	177.1	186.1	193.7	197.6	201.5
199									5.7	-5.5	3.3	6.9	11.1	15.8	15.2	19.2	23.4	30.9	31.7	28.4
200									10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%

MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
201	Computed value of oper								213.7	229.5	258.1	280.7	302.0	321.2	337.7	356.4	373.0	387.0	395.0	403.0
202	assets (at year start)																			
203	Computed equity value								98.2	113.5	130.3	142.3	153.2	162.8	170.7	179.3	186.8	193.3	197.5	201.5
204	(at year start)																			
205	Computed equity value at																			
206	the start of year 1 inclu-																			
207	ding dividends at the								101.1											
208	end of year 0																			
209																				
210	Desired E/(D+E) year 12								50.0%											
211	Resulting E/(D+E) year 12								50.0%											
212	Result E/(D+E) all years								46.0%	49.5%	50.5%	50.7%	50.7%	50.7%	50.6%	50.3%	50.1%	50.0%	50.0%	50.0%
213	Drive to 0 by Goal Seek								0.00											
214	(vary year 1 Book value																			
215	target for financial																			
216	strength 1183!																			
217																				
218	2. Discounted Dividends,																			
219	constant WACC weights																			
220																				
221	Dividends (at year end)								0.0	0.0	7.1	9.9	12.8	15.9	16.4	18.7	21.0	24.4	25.1	23.7
222	Cost of equity								14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%
223	Computed equity value																			
224	(at year start)								98.5	113.0	129.7	141.7	152.7	162.4	170.5	179.2	186.8	193.4	197.5	201.5
225																				
226	3. Adjusted Present Value																			
227																				
228	Interest-bearing debt																			
229	(at year start)								115.5	116.0	127.7	138.4	148.8	158.4	166.9	177.1	186.1	193.7	197.6	201.5
230	Free cash fl (at year end)								5.7	-5.5	3.3	6.9	11.1	15.8	15.2	19.2	23.4	30.9	31.7	28.4
231	Tax shield (at year end)								3.9	4.0	4.4	4.7	5.1	5.4	5.7	6.1	6.4	6.6	6.8	6.9
232	Opportunity cost of cap								11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%
233	Computed value of oper																			
234	assets (at year start)								160.4	173.6	199.5	219.7	238.6	255.6	269.8	286.3	300.8	312.7	318.6	324.3
235	not including tax shields																			
236	Present value of tax								53.4	55.7	58.3	60.8	63.2	65.5	67.8	70.0	72.2	74.3	76.5	78.7
237	shields (at year start)																			
238	Computed value of oper																			
239	assets (at year start)								213.8	229.3	257.8	280.5	301.8	321.1	337.6	356.4	373.0	387.0	395.0	403.0
240	including tax shields																			
241	Computed equity value								98.3	113.3	130.1	142.1	153.0	162.6	170.6	179.2	186.8	193.3	197.5	201.5
242	(at year start)																			
243																				
244	4. Disc't Cash Flow, non-																			
245	constant WACC weights																			
246																				
247	Interest-bearing debt																			
248	(at year start)								115.5	116.0	127.7	138.4	148.8	158.4	166.9	177.1	186.1	193.7	197.6	201.5
249	Free cash fl (at year end)								5.7	-5.5	3.3	6.9	11.1	15.8	15.2	19.2	23.4	30.9	31.7	28.4
250	WACC																			10.05%

MCKAY VALUATION BY DISCOUNTED CASH FLOW, DISCOUNTED DIVIDENDS, ADJUSTED PRESENT VALUE; TWO WACC CALCULATIONS (FILE MCK.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
251	Opportunity cost of cap								11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	11.76%	
252	Computed value of oper																			
253	assets (at year start)								213.8	229.3	257.8	280.5	301.8	321.1	337.6	356.4	373.0	387.0	395.0	403.0
254	Computed equity value								98.3	113.3	130.1	142.1	153.0	162.6	170.6	179.2	186.8	193.3	197.5	201.5
255	(at year start)																			
256																				
257	Desired E/(D+E) year 12								46.0%	49.4%	50.5%	50.7%	50.7%	50.7%	50.5%	50.3%	50.1%	50.0%	50.0%	50.0%
258	Resulting E/(D+E)								9.91%	10.03%	10.06%	10.07%	10.07%	10.07%	10.06%	10.06%	10.05%	10.04%	10.04%	10.04%
259	Resulting WACC								15.26%	14.81%	14.69%	14.66%	14.66%	14.66%	14.68%	14.71%	14.73%	14.75%	14.74%	14.74%
260	Resulting cost of equity																			
261																				
262	5. Discounted Div's, non-																			
263	constant WACC weights																			
264																				
265	Dividends (at year end)								0.0	0.0	7.1	9.9	12.8	15.9	16.4	18.7	21.0	24.4	25.1	23.7
266	Cost of equity								15.26%	14.81%	14.69%	14.66%	14.66%	14.66%	14.68%	14.71%	14.73%	14.75%	14.74%	14.74%
267	Computed equity value																			
268	(at year start)								98.3	113.3	130.1	142.1	153.0	162.6	170.6	179.2	186.8	193.3	197.5	201.5

MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	TABLE 1. HISTORICAL INCOME STATEMENTS																		
1																			
2																			
3	<i>Income statement</i>	-6	-5	-4	-3	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
4	Revenues	197.6	222.3	272.3	299.5	418.9	505.4	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1229.4	1266.3	1304.3
5	Operating expenses	-175.4	-205.8	-249.6	-274.7	-383.6	-467.4	-538.8	-621.5	-691.4	-762.0	-831.9	-899.7	-963.8	-1022.5	-1074.2	-1106.5	-1139.7	-1173.9
6	Depreciation	-12.8	-9.3	-11.2	-13.0	-17.7	-26.4	-28.7	-32.0	-36.6	-40.2	-43.8	-47.2	-50.4	-54.3	-57.9	-61.2	-63.3	-64.5
7	Operating income	9.4	7.2	11.5	11.8	14.5	11.6	31.1	37.1	40.3	44.4	48.6	52.7	56.7	59.3	61.5	61.8	63.3	65.9
8	Interest income	0.0	0.0	0.0	0.0	0.7	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	Interest expense	-0.1	-0.4	-0.8	-1.0	-3.4	-10.1	-9.4	-10.1	-11.3	-12.3	-13.2	-14.1	-14.8	-15.6	-16.4	-17.0	-17.3	-17.7
10	Earnings before taxes	9.3	6.8	10.7	10.8	14.2	2.1	21.7	27.0	28.9	32.1	35.4	38.7	41.9	43.7	45.1	44.8	46.0	48.2
11	Revaluation of deferred income taxes							0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	Income taxes	-3.3	-2.4	-3.8	-4.2	-6.1	-0.7	-8.5	-10.5	-11.3	-12.5	-13.8	-15.1	-16.3	-17.0	-17.6	-17.5	-17.9	-18.8
13	Net income	6.0	4.4	6.9	6.6	7.0	8.1	13.3	16.5	17.7	19.6	21.6	23.6	25.6	26.7	27.5	27.3	28.1	29.4
14	Statement of common equity																		
15	Beginning common equity	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
16	Net income																		
17	Common dividends																		
18	New share issue																		
19	Ending common equity	67.1	68.6	89.4	92.2	93.4	104.7	110.2	124.0	134.0	143.5	152.4	160.5	170.6	179.3	186.3	189.1	192.1	197.8
20	TABLE 2. HISTORICAL BALANCE SHEETS																		
21																			
22																			
23																			
24	Operating cash	4.0	4.4	5.4	6.0	6.0	8.4	10.1	11.7	13.5	15.0	16.4	17.9	19.3	20.6	21.8	22.8	23.4	24.1
25	Excess marketable securities *	10.9	3.0	20.5	10.3	0.0	5.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	Trade receivables	17.9	24.4	33.0	33.3	43.9	49.8	57.7	67.7	77.8	86.3	94.8	103.2	111.3	118.9	125.7	131.7	135.2	138.8
27	Other receivables	1.5	2.0	2.7	2.7	6.2	4.9	5.7	6.5	7.5	8.3	9.1	9.9	10.7	11.4	12.1	12.7	13.0	13.4
28	Inventories	1.9	2.1	2.8	2.5	9.0	10.9	11.9	9.5	10.9	12.1	13.3	14.5	15.6	16.7	17.6	18.5	19.0	19.5
29	Prepaid expenses	4.3	5.1	5.3	6.0	2.4	4.4	5.0	9.5	11.0	12.2	13.4	14.5	15.7	16.7	17.7	18.5	19.0	20.1
30	Current assets	40.5	41.0	69.7	60.8	67.5	84.2	90.4	104.9	120.7	133.8	147.1	160.1	172.6	184.3	195.0	204.2	209.7	215.3
31	Gross property, plant and equipment	100.0	117.7	128.2	155.6	204.7	272.5	297.6	334.2	378.9	417.7	454.9	489.8	521.4	555.8	586.6	613.0	630.4	645.6
32	Accumulated depreciation	-37.7	-42.3	-48.7	-56.5	-71.9	-86.9	-103.4	-121.8	-140.4	-159.5	-178.3	-196.5	-213.7	-229.4	-243.8	-256.4	-266.9	-275.0
33	Net property, plant																		
34																			
35																			
36																			
37																			
38																			
39																			
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MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
51	and equipment	62.3	75.4	79.5	99.1	132.8	185.6	194.2	212.4	238.6	258.2	276.6	293.3	307.8	326.4	342.9	356.6	363.6	370.6	381.7
52																				
53	Total assets	102.8	116.4	149.2	159.9	200.3	269.8	284.6	317.4	359.3	392.0	423.6	453.3	480.4	510.7	537.8	560.8	573.2	585.9	603.5
54																				
55																				
56	Short-term debt	0.3	0.8	0.9	1.5	11.5	12.5	20.7	17.2	19.5	21.9	23.7	25.5	27.0	28.4	30.0	31.3	32.4	33.0	33.7
57	Accounts payable	7.3	11.0	11.9	10.5	14.2	16.2	18.9	23.7	27.3	30.4	33.5	36.6	39.6	42.4	45.0	47.3	48.7	50.1	51.6
58	Other current liabilities	13.9	13.5	18.2	18.5	21.4	27.8	28.8	38.0	43.8	48.7	53.7	58.7	63.4	68.0	72.1	75.7	78.0	80.4	82.8
59																				
60	Total current liabilities	21.5	25.3	31.0	30.5	47.1	56.5	68.4	78.9	90.7	101.1	110.9	120.7	130.0	138.7	147.0	154.3	159.1	163.5	168.1
61																				
62	Long term debt	5.5	11.5	16.2	21.7	40.2	90.6	86.2	97.5	109.5	118.5	127.3	135.1	141.8	149.8	156.5	162.2	165.1	168.5	173.8
63																				
64	Deferred income taxes	8.7	11.0	12.6	15.5	19.6	25.1	25.3	30.8	35.1	38.5	41.9	45.1	48.0	51.6	55.0	58.0	59.9	61.9	63.7
65																				
66	Common stock	4.6	4.9	23.6	23.6	23.6	23.6	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3	29.3
67	Retained earnings	62.5	63.7	65.8	68.6	69.8	74.0	75.4	80.8	94.6	104.6	114.1	123.1	131.2	141.2	149.9	157.0	159.8	162.7	168.5
68																				
69	Total common equity	67.1	68.6	89.4	92.2	93.4	97.6	104.7	110.2	124.0	134.0	143.5	152.4	160.5	170.6	179.3	186.3	189.1	192.1	197.8
70																				
71	Total liabilities and equity	102.8	116.4	149.2	159.9	200.3	269.8	284.6	317.4	359.3	392.0	423.6	453.3	480.4	510.7	537.8	560.8	573.2	585.9	603.5
72																				
73																				
74	Invested capital	70.7	88.9	98.6	120.6	164.7	220.0	236.9	255.7	288.1	312.8	336.4	358.1	377.3	400.4	420.7	437.8	446.5	455.4	469.1
75	Debt/invested capital	-7.2%	10.5%	-3.4%	10.7%	31.4%	44.2%	45.1%	44.9%	44.8%	44.9%	44.9%	44.9%	44.7%	44.5%	44.3%	44.2%	44.2%	44.2%	44.2%
76	NOPLAT/invested capital																			
77	(ROIC)		8.7%	9.6%	9.2%	8.9%	8.1%	3.2%	9.9%	9.9%	9.3%	9.4%	9.5%	9.5%	9.8%	9.6%	9.4%	9.0%	9.0%	9.1%
78	* direct forecast																			
79																				
80																				
81																				
TABLE 3. HISTORICAL FREE CASH FLOW																				
82	Free cash flow	-5	-4	-3	-2	-1	0		1	2	3	4	5	6	7	8	9	10	11	12
83																				
84																				
85	Revenues	222.3	272.3	299.5	350.0	418.9	505.4	598.6	690.6	768.2	846.7	924.4	999.7	1070.9	1136.1	1193.6	1229.4	1266.3	1304.3	1340.3
86	Operating expenses	-205.8	-249.6	-274.7	-320.5	-383.6	-467.4	-538.8	-621.5	-691.4	-762.0	-831.9	-899.7	-963.8	-1022.5	-1074.2	-1106.5	-1139.7	-1173.9	-1209.9
87	Depreciation	-9.3	-11.2	-13.0	-15.0	-17.7	-26.4	-28.7	-32.0	-36.6	-40.2	-43.8	-47.2	-50.4	-54.3	-57.9	-61.2	-63.3	-64.5	-65.7
88																				
89	EBIT	7.2	11.5	11.8	14.5	17.6	11.6	31.1	37.1	40.3	44.4	48.6	52.7	56.7	59.3	61.5	61.8	63.3	65.9	68.5
90	Taxes on EBIT	-2.6	-4.1	-4.6	-6.0	-7.4	-4.4	-12.1	-14.5	-15.7	-17.3	-19.0	-20.6	-22.1	-23.1	-24.0	-24.1	-24.7	-25.7	-26.7
91	Revaluation of deferred																			
92	income taxes																			
93	Change in deferred																			
94	income taxes	2.3	1.6	2.9	4.1	5.5	0.2	5.5	4.3	3.5	3.4	3.4	3.2	2.9	3.6	3.4	3.0	1.9	2.0	1.9
95																				
96	NOPLAT	6.9	9.0	10.1	12.6	15.7	7.4	24.4	26.9	28.0	30.5	32.8	35.1	38.2	39.6	40.5	39.6	40.6	42.1	43.6
97	Add back depreciation	9.3	11.2	13.0	15.0	17.7	26.4	28.7	32.0	36.6	40.2	43.8	47.2	50.4	54.3	57.9	61.2	63.3	64.5	65.7
98																				
99	Gross cash flow	16.2	20.2	23.1	27.6	33.4	33.8	53.2	58.9	64.6	70.7	76.7	82.3	88.6	93.8	98.4	100.8	103.9	106.6	109.3
100																				

MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
101	Change in working capital (+ if increase)		5.1	5.6	2.4	10.4	2.5	8.3	0.5	6.3	5.2	5.1	5.0	4.8	4.4	3.9	3.3	1.8	1.8	2.5
103	Capital expenditures		22.4	15.3	32.6	48.7	35.0	35.0	47.0	58.1	56.2	58.6	60.5	61.7	69.0	70.7	71.6	68.1	70.4	75.6
104	(+ if positive)		27.5	20.9	35.0	59.1	73.0	43.3	47.5	64.4	61.3	63.8	65.5	66.5	73.4	74.6	75.0	69.9	72.2	78.2
106	Gross investment		-11.3	-0.7	-11.9	-31.5	-9.5	-9.5	5.7	-5.5	3.3	6.9	11.1	15.8	15.2	19.2	23.4	30.9	31.7	28.4
107	Free cash flow																			
109	Financial cash flow																			
110	Incr(+)/Deer(-) in excess marketable securities		-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
112	After-tax interest																			
113	income (-)																			
114	Incr(+)/Deer(+)		-7.9	17.5	-10.2	-10.3	5.8	-5.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
115	and long-term debt																			
116	expense (+)		0.0	0.0	0.0	-0.5	-0.4	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
117	Common dividends (+)		-6.5	-4.8	-6.1	-28.5	-51.4	-3.8	-7.9	-14.3	-11.3	-10.6	-9.6	-8.2	-9.4	-8.3	-7.0	-4.0	-4.0	-6.0
118	Incr(+)/Deer(+)		0.2	0.5	0.6	2.1	2.5	6.2	5.7	6.1	6.9	7.5	8.1	8.6	9.0	9.5	10.0	10.3	10.6	10.8
119	common stock		3.2	4.8	3.8	5.8	3.9	0.0	7.8	2.6	7.7	10.1	12.7	15.5	15.5	18.0	20.5	24.5	25.1	23.7
120	Financial cash flow		-0.3	-18.7	0.0	0.0	0.0	-5.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
121	Effective tax rate on EBIT																			
122																				
123																				
124																				
125																				
126																				
127																				
128																				
129																				
130																				
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MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
151	Property, Plant and Equipment (PPE)																			
152	PPE economic life (n)																			
153	F_sub_g (see Sect 6)																			10
154	F_sub_c (see Sect 6)																			10.0000
155	Real PPE/rev's (K)																			8.7861
156	M (see Section 6)																			0.580
157	H (see Section 6)																			0.510
158	Net PPE/revenues	31.5%	33.9%	29.2%	33.1%	37.9%	44.3%	38.4%	35.5%	34.5%	33.6%	32.7%	31.7%	30.8%	30.5%	30.2%	29.9%	29.6%	29.3%	29.3%
159	Depreciation/net PPE *	14.9%	14.9%	16.4%	15.1%	15.1%	13.3%	14.2%	14.8%	15.1%	15.3%	15.6%	15.8%	16.1%	16.4%	16.6%	16.9%	17.2%	17.4%	17.4%
160	Retirements/net PPE *	7.5%	6.4%	6.5%	6.5%	-0.4%	2.0%	5.3%	5.3%	6.3%	7.3%	8.3%	9.3%	10.3%	11.2%	12.2%	13.2%	14.2%	15.2%	15.2%
161	Net PPE/revenues curve parameter (a)																			
162	Inflation factors								1.344	1.305	1.267	1.230	1.194	1.159	1.126	1.093	1.061	1.030	-0.00319	
163	Resulting real gross PPE																			1.000
164	Required real gross PPE																			734.4
165	Drive to 0 by Goal Seek (vary S163)!																			734.4
166	Taxes																			
167	Tax rate	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%
168	PPE tax life (g)																			5
169	J (see Section 8)																			2.160
170	Timing diffrs/net PPE	35.8%	37.4%	40.6%	40.1%	37.8%	34.7%	33.4%	37.1%	37.7%	38.3%	38.8%	39.4%	40.0%	40.5%	41.1%	41.7%	42.2%	42.8%	42.8%
171	Interest rate items																			
172	Real borrowing rate								5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%	5.60%
173	Nominal borrowing rate								8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%	8.77%
174	Real cost of equity								11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%	11.40%
175	Nominal cost of equity								14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%	14.74%
176	Implied borrowing rate																			
177	This year's short-term/last year's long-term debt								20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
178	* last year's net PPE																			
179	Macro shortcut: Ctrl+Shift+I																			
180	VALUE CALCULATIONS																			
181	1	2	3	4	5	6	7	8	9	10	11	12								
182	Interest-bearing debt (at year start)																			
183	Free cash fl (at year end)																			
184	WACC																			
185	Computed value of oper assets (at year start)																			
186	Computed equity value																			

MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
201	(at year start)								106.9	114.8	129.0	140.4	151.0	160.6	168.8	178.2	186.5	193.5	197.5	201.5
202	Computed equity value at the start of year 1 including div's or new share issue at the end of year 0																			
203									101.1											
204																				
205																				
206																				
207	Desired E/(D+E)								50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
208	Resulting E/(D+E)								50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
209	Absolute deviations								0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
210	Drive to zero by Solver								0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
211	(vary H62:S62)								0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
212																				
213	2a. Economic Value																			
214	Added (first variant)																			
215																				
216	Interest-bearing debt																			
217	(at year start)								106.9	114.8	129.0	140.4	151.0	160.6	168.8	178.2	186.5	193.5	197.5	201.5
218	Invested capital (at year start)								236.9	255.7	288.1	312.8	336.4	358.1	377.3	400.4	420.7	437.8	446.5	455.4
219	NOPLAT (at year end)								24.4	26.9	28.0	30.5	32.8	35.1	38.2	39.6	40.5	39.6	40.6	42.1
220	WACC								10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%
221	Economic value added								0.6	1.2	-0.9	-1.0	-1.0	-0.9	0.3	-0.7	-1.8	-4.3	-4.2	-3.7
222	Computed value of econ value added (at year st)								-23.2	-26.2	-30.0	-32.1	-34.4	-36.9	-39.7	-44.0	-47.7	-50.8	-51.5	-52.4
223	assets (at year start)								213.7	229.5	258.1	280.7	302.0	321.2	337.7	356.4	373.0	387.0	395.0	403.0
224	Computed equity value (at year start)								106.9	114.8	129.0	140.4	151.0	160.6	168.8	178.2	186.5	193.5	197.5	201.5
225	Computed value of oper assets (at year start)								Computed value of operating assets at the horizon also (cf. Skogsvik p. 19)											
226									Free cash flow at the end of the first post-horizon year also (cf. Skogsvik p. 19)											
227																				
228																				
229																				
230																				
231																				
232	2b. Economic Value																			
233	Added (second variant)																			
234																				
235	Interest-bearing debt																			
236	(at year start)								106.9	114.8	129.0	140.4	151.0	160.6	168.8	178.2	186.5	193.5	197.5	201.5
237	Invested capital (at year start)																			
238	start, deduct (def) taxes								211.6	224.9	253.0	274.3	294.5	313.0	329.4	348.8	365.8	379.8	386.6	393.6
239	EBIT minus taxes on EBIT																			
240	WACC								19.0	22.6	24.6	27.1	29.7	32.2	34.6	36.2	37.5	37.7	38.6	40.2
241	Economic value added								10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%	10.05%
242	Computed value of econ value added (at year st)								-2.3	0.0	-0.9	-0.5	0.1	0.7	1.5	1.2	0.7	-0.5	-0.2	0.7
243	assets (at year start)								2.1	4.6	5.0	6.4	7.5	8.2	8.3	7.6	7.2	7.2	8.4	9.4
244	Computed value of oper assets (at year start)								213.7	229.5	258.1	280.7	302.0	321.2	337.7	356.4	373.0	387.0	395.0	403.0
245																				
246	Computed equity value (at year start)								106.9	114.8	129.0	140.4	151.0	160.6	168.8	178.2	186.5	193.5	197.5	201.5
247									Computed value of operating assets at the horizon also (cf. Skogsvik p. 19)											
248									Free cash flow at the end of the first post-horizon year also (cf. Skogsvik p. 19)											
249																				
250																				

MCKAY VALUATION, DIFFERENT FINANCING POLICY, FIVE VALUATION MODELS (FILE MCK_EXT.XLS)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
251																				
252	3. Adjusted Present Value																			
253																				
254	Interest-bearing debt																			
255	(at year start)																			
256	Free cash fl (at year end)																			
257	Tax shield (at year end)																			
258	Opportunity cost of cap																			
259	Computed value of oper																			
260	assets (at year start)																			
261	not including tax shields																			
262	Present value of tax																			
263	shields (at year start)																			
264	Computed value of oper																			
265	assets (at year start)																			
266	including tax shields																			
267	Computed equity value																			
268	(at year start)																			
269																				
270	4. Discounted Dividends																			
271																				
272	Dividends (at year end)																			
273	Cost of equity																			
274	Computed equity value																			
275	(at year start)																			
276																				
277	5. Abnormal Earnings																			
278																				
279	Book equity value																			
280	(at year start)																			
281	Net income (at year end)																			
282	Cost of equity																			
283	Abnormal earnings																			
284	Computed value of abn																			
285	earnings (at year start)																			
286	Computed equity value																			
287	(at year start)																			
288																				
289																				

Computed equity value at horizon also (cf. Skogsvik (6) p. 8)
 Dividends at the end of the first post-horizon year also (cf. Skogsvik footnote 7 p. 8)