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A Tutorial on Residual Income Valuation and Value Added Valuation

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1. Introduction

The challenge of using accounting numbers for valuation purposes has tempted accounting researchers and professional financial analysts over the years. The choice and measurement of suitable accounting numbers, as well as the specification of the linkage between accounting numbers and stock market prices, have constituted important issues. A wide array of valuation models have been suggested over time, including simple models based only on measures of current earnings, as well as elaborate simulation models based on a multitude of accounting numbers. From a methodological point of view, the proposed valuation models can be divided into two main groups:

- Valuation models that are directly based on the statistical association between accounting numbers and stock market prices.

- Valuation models that are deduced from the theory of capital value.

Models of the first kind – *statistical valuation models* – often hinge on some simplified assumption about the relationship between accounting numbers and stock market prices, e.g. the simple mathematical relationship of a P/E-ratio valuation model. Hence, such models are often viewed as particularly easy to use. The price of this simplicity is typically, however, deficiencies in the modelling logic. Furthermore, statistical valuation models can only be estimated when there is some empirical market data to be observed. A prerequisite of such models is then that observed stock market prices are 'correct', i.e. that the valuation analysis which investors actually engage in lead to prices that fully reflect all available information. This assumption corresponds to the well-known hypothesis of 'semi-strong market efficiency '.[1] Whether this hypothesis is empirically valid is not clear, however.

[1] The market is said to be 'semi-strong efficient' if all public information, including publicly available accounting information, is fully reflected in stock market prices.

Models of the second kind – *deduced valuation models* – have been subject to an increased interest in the academic research since the beginning of the 1990's.[2] These models do not depend on any assumption about stock market prices being efficient in the semi-strong sense. In general, they constitute a good foundation for the specification of relationships between accounting numbers and stock market prices. However, statistical problems – in particular concerning the prediction of valuation relevant accounting numbers – cannot be avoided in these models. Such problems can often be analytically isolated though, whereby guidelines for the estimation of statistical forecasting models can be provided.

In the following, two deduced valuation models will be specified and discussed – a *'residual income'* valuation model (section 2) and a '*value added'* valuation model (section 3). Both models are based on a modelling logic where capital values are determined as the sum of an accounting book measure of capital, the present value of expected future abnormal profitability, and the present value of expected goodwill/ badwill at some horizon point in time. Strengths and weaknesses of the two valuation models will be discussed in section 4.

2. Residual Income Valuation

2.1 Model specification

The underlying value attribute of the residual income valuation model is the net dividends being paid to the shareholders of the company, i.e. expected dividends less any capital contributions. In accordance with the theory of capital value, the value of owners' equity is then obtained as the present value of future expected (net-)dividends, as expressed in (1):

$$(1) \qquad V_0 = \sum_{t=1}^{\infty} \frac{D_t - N_t}{(1+\rho)^t}$$

[2] In the US, professors James Ohlson (New York University), George Feltham (University of British Columbia) and Stephen Penman (Columbia University) have been pioneers in this renaissance of fundamental valuation analysis (cf. Ohlson, 1995, Feltham & Ohlson, 1995, Penman, 1992, and Skogsvik, 1994).

where: V_0 = capital value of owners' equity, determined ex dividend and including any new issue of share capital at time t = 0

D_t = expected total dividend paid to the shareholders of the company, where t denotes time of payment

N_t = expected new issue of share capital to the company, where t denotes time of payment

ρ = required rate of return on owners' equity (= cost of equity capital)

The valuation function in (1) is consistent with the idea that a company – in the absence of any indications of the opposite – is expected to 'live forever'. For reasons typically concerned with forecasting issues, a finite horizon point in time is often introduced in this function. Thus expression (2) is obtained.

$$(2) \qquad V_0 = \sum_{t=1}^{T} \frac{D_t - N_t}{(1+\rho)^t} + \frac{V_T}{(1+\rho)^T}$$

where: V_T = expected capital value of owners' equity at the horizon point in time t = T (ex dividend and including any new issue of share capital at time t = T)

Assuming that the 'clean surplus relation of accounting' holds in each period (i.e. that net income, dividends and new issues of share capital explain changes in the book value of owners' equity), (net-)dividends to company shareholders can be reexpressed as follows:

$$(3) \qquad D_t - N_t = B_{t-1} + I_t - B_t = B_{t-1} \cdot R^*_{E,t} - \left(B_t - B_{t-1}\right)$$

where: B_t = book value of owners' equity, determined ex dividend and including any new issue of share capital at time t

I_t = accounting net income, accrued in period t

$R^*_{E,t} \equiv I_t/B_{t-1}$ = book return on owners' equity, accrued in period t

The clean surplus relation of accounting hence implies that the (net-)dividend being paid at the end of some period, coincides with the difference between the accounting net income and the change in the book value of owners' equity during the period. With reference to the book return on owners' equity, net income can be written as $B_{t-1} \cdot R^*_{E,t}$ and thus the (net-)dividend can be expressed as $B_{t-1} \cdot R^*_{E,t} - (B_t - B_{t-1})$.

The difference between the book return and the required rate of return on owners' equity can be viewed as a simple measure of 'residual' book return. Evidently $R^*_{E,t}$ can be rewritten as $\rho + (R^*_{E,t} - \rho)$, meaning that (3) can be restated as follows:

$$(3') \qquad D_t - N_t = B_{t-1}\left[\rho + \left(R^*_{E,t} - \rho\right)\right] - (B_t - B_{t-1})$$

If future (net-)dividends in the valuation function (2) are rewritten in accordance with (3'), the following expression – a 'residual income' valuation model – can be derived:

$$(4) \qquad V_0 = B_0 + \sum_{t=1}^{T} \frac{B_{t-1}(R^*_{E,t} - \rho)}{(1+\rho)^t} + \frac{B_T(V_T / B_T - 1)}{(1+\rho)^T}$$

The valuation function in (4) shows that the value of owners' equity is calculated as the sum of the following components:

- Accounting book value – the book value of owners' equity, determined ex dividend and including any new issue of share capital, at the valuation point in time. (The accounting book value is denoted B_0 in the valuation function.)

- Present value of the expected residual income until the horizon point in time. The residual income is calculated for each period as the product of the book value of owners' equity at the beginning of the period and the difference between $R^*_{E,t}$ and ρ. (The present value of the expected residual income is written as $\sum_{t=1}^{T} B_{t-1}(R^*_{E,t} - \rho)/(1+\rho)^t$ in the valuation function.)

- <u>Present value of the expected goodwill/badwill of owners' equity</u> at the horizon point in time. (In the valuation function this value is expressed as $B_T(V_T/B_T - 1)/(1 + \rho)^T$. Note that the goodwill/badwill of owners' equity at time t = T in principle is equal to the present value of future expected residual income *after* the horizon point in time.)

The valuation function in (4) is of particular interest as it is conditioned only on the assumption of the clean surplus relation of accounting.[3] This means that the valuation function is as applicable to historical cost accounting as to inflation or current cost accounting, as long as the accounting is done in compliance with the clean surplus relation. It should also be noted that this relationship between accounting numbers and capital values has been known for a long period of time; early references go back to Preinreich (1938) and Edwards & Bell (1961).

Assuming a constant annual growth of the book value of owners' equity until time t = T, (4) can be rewritten somewhat. If the future growth of owners' equity is denoted δ, it follows that $B_{t-1} = B_0(1 + \delta)^{t-1}$ and $B_T = B_0(1+\delta)^T$ in (4), and a new valuation function is obtained:

$$(5) \qquad V_0 = B_0\left[1 + \sum_{t=1}^{T} \frac{(1+\delta)^{t-1}(R^*_{E,t} - \rho)}{(1+\rho)^t} + \frac{(1+\delta)^T (V_T/B_T - 1)}{(1+\rho)^T}\right]$$

Of course, the interpretation of the expression above coincides with the interpretation of the valuation function in (4). The additional assumption has made it possible to write B_0 as a separate factor though, and rewriting (5) somewhat an expression of relative goodwill/badwill of owners' equity at the valuation point in time (i.e. $V_0/B_0 - 1$), can easily be derived. (Note that the valuation function in (5) can only be used when the expected future growth of owners' equity to the horizon point in time is a constant.)

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[3] To be more precise, it is actually assumed that the present value of the expected value of future deviations from the clean surplus relation is zero, an assumption which can hold even if the clean surplus relation is not fulfilled in all future periods.

2.2 Assessments of valuation parameters

In order for the valuation models in (4) or (5) to be practically useful, assessments of the model parameters should be possible to make in a reasonably simple and robust way. In this regard, the book value of owners' equity at present (B_0) is unproblematic – with access to the latest financial report of the company this measure is easily obtained.[4] (As noted above, alternative measurement principles – e.g. concerning the valuation of immaterial assets or deferred taxes – may be used in determining B_0, as long as the clean surplus relation of accounting is maintained). The required rate of return on owners' equity ρ, on the other hand, is associated with considerable theoretical and methodological difficulties – let us, however, in the present context assume that this parameter is known. The following prediction problems then remain:

- What is the expected future growth of owners' equity to the horizon point in time t = T?
- What is the expected future book return on owners' equity ($R^*_{E,t}$) to the horizon point in time t = T?
- What is the expected relative goodwill/badwill of owners' equity at some 'appropriately' chosen horizon point in time (V_T/B_T - 1)?

How the above prediction problems should be solved is not obvious. To an investor with no access to 'inside' management information, historical financial statements for the company are expected to be important. As a first, 'naive', solution to the prediction problems, the following guidelines can then be suggested.

The last prediction problem stated above – i.e. the estimation of an expected value of (V_T/B_T -1) – is typically a reasonable starting point. In principle, the value of goodwill/badwill of owners' equity is explained by 'business goodwill' and some

[4] Note that B_0 is measured <u>after</u> any dividend has been paid to the shareholders and/or any new issue of share capital has been paid to the company at time t=0.

accounting measurement bias. The business goodwill – or badwill – depends on whether the expected 'true' return on owners' equity exceeds – or falls below – the market's required rate of return on owners' equity. In principle, a positive business goodwill occurs when the internal rate of return on current and future expected business projects exceeds the required rate of return on these projects. The accounting measurement bias, on the other hand, is explained by discrepancies between a 'true and fair' matching of company revenues and costs, and the actual matching that takes place in the accounting reports. If, for example, conservative accounting principles are applied in the financial statements, the accounting measurement bias typically causes $(V_T/B_T - 1) > 0$ (even in situations when the company business goodwill = 0).

In a prediction context, making a distinction between business goodwill and accounting measurement bias is important since the business goodwill can be expected to diminish over time, while the measurement bias can be expected to remain. For example, increased business competition and higher wage demands and/or other input price increases, are typical forces in a market economy causing a positive business goodwill to evaporate in a future 'steady state' equilibrium.[5]

The first step would hence be to make an assessment of the horizon point in time $t = T$, such that the business goodwill can be expected to be negligible at this point in time. Consequently, only the accounting measurement bias would remain to be estimated. This is by no means a trivial task – a thorough knowledge of company characteristics, macroeconomic conditions and accounting measurement principles, is typically required. Current cost accounting values of assets and liabilities – preferably with capitalized values of intangible assets – can provide guidance in the assessment of the measurement bias. (If the current cost value of owners' equity at $t = 0$ is denoted $B_0^{(C)}$, a relative accounting measurement bias of owners' equity at this date can be estimated as $\left(B_0^{(C)} / B_0 - 1\right)$. Whether this value would constitute a valid estimate of the

[5] Technically speaking, the present value of expected business goodwill approaches 0 when T is a 'sufficiently' large number.

relative measurement bias at the horizon point in time t = T is not obvious – a necessary condition for this to be the case is that the company asset structure at time t = 0 and t = T is about the same. Furthermore, relative price changes prior to t = T are required to be about the same as current and past relative price changes.)[6]

If the relative measurement bias of owners' equity ($V_{T+t}/B_{T+t} - 1$) coincides with ($V_T/B_T - 1$) from time T and onwards, and the growth of owners' equity is constant after time t = T, the expected book return on owners' equity after the horizon point in time can be determined as follows:[7]

$$(6) \qquad R^*_{E,T+1} = R^*_{E,T+2} = \cdots = R^*_{E,T+\infty} = \rho + (V_T/B_T - 1)(\rho - \delta')$$

where: δ' = expected annual growth of owners' equity after time t = T

In accordance with (6), the expected book return on owners' equity after the horizon point in time is a constant. Note furthermore that the book return coincides with the required rate of return ρ if the accounting measurement bias is 0 (i.e. $V_T/B_T - 1 = 0$). Also, if the difference between the required rate of return and the expected growth of owners' equity (i.e. $\rho - \delta'$) is negligible, the discrepancy between the book return and the required rate of return will be insignificant.

The next task would be to address the second prediction problem put forth above – i.e. to predict the book return on owners' equity for the periods t = 1, t = 2,... to t = T. One way to deal with this problem would be to forecast a value of the book return for next

[6] If these conditions are not fulfilled, a horizon value of owners' equity based on current cost accounting principles should rather be predicted. A thorough discussion of this issue, as well as suggested estimation procedures for determining the accounting measurement bias, can be found in Runsten (1998) (especially pp. 57-87 and pp. 140-151).

[7] Cf. pp. 24-25 in Skogsvik (1998). The relationship in (6) can be solved for V_T:

$$(6') \quad V_T = B_T(R^*_{E,T+1} - \delta')/(\rho - \delta')$$

If the clean surplus relation of accounting holds, the numerator on the right-hand side of (6') coincides with the expected dividend at time T+1. Since the expected book return on owners' equity is a constant, future dividends and owners' equity are expected to grow at the same rate (=δ'). Thus, if $\rho > \delta'$, (6) is consistent with the valuation of owners' equity according to the 'Gordon's growth model'.

year ($R^*_{E,1}$) and thereafter assume a gradual process of adjustment to the long term, 'steady state', book return $R^*_{E,T+1}$ (determined in accordance with (6) above). The assessment of $R^*_{E,1}$ can be based on different sources of information – for example the company book return on owners' equity in previous periods, or earnings forecasts made by the management or professional financial analysts. In theory the time series adjustment from $R^*_{E,1}$ to $R^*_{E,T+1}$ is affected by periodic changes in the company business goodwill and the accounting measurement bias. In the absence of other relevant information, a simple approach here is to suggest a linear gradual change from $R^*_{E,1}$ to $R^*_{E,T+1}$ over the time interval t = 1 to t = T + 1; i.e.:

(7) $$R^*_{E,t} = R^*_{E,1} + (t-1)\left(R^*_{E,T+1} - R^*_{E,1}\right)/T$$
$$\text{for } 2 \le t \le T$$

An alternative adjustment procedure would be to change $R^*_{E,t}$ in some 'stepwise' fashion over time – as an extreme it might even be assumed that $R^*_{E,t} = R^*_{E,1}$ for all years t = 1, t = 2,... to t = T.

The remaining prediction problem is concerned with the assessment of the expected future growth of owners' equity up to the horizon point in time. Having access to forecasts of the future business growth made by the management or financial analysts, it can be reasonable to make forecasts of growth in owners' equity *per se*.[8] When dealing with listed companies, it might also be useful to consider the assessment of a 'robust' dividend policy, e.g. in the sense that the dividend payout ratio, $D_t/Income_t = pr_t$, or the dividend share ratio, $D_t/B_{t-1} = ps_t$, is stable over time. Assuming that the clean surplus relation of accounting holds, and disregarding any new issues of share capital, the following expressions will then hold:

[8] Cf. the model of financial planning which is discussed in Johansson (1998), especially pp. 87-91 and 129-134.

(8.a) $$\frac{B_t}{B_{t-1}} - 1 = (1 - pr_t)R^*_{E,t} \qquad \text{where } pr_t = pr \quad \text{for } 1 \le t \le T$$

(8.b) $$\frac{B_t}{B_{t-1}} - 1 = R^*_{E,t} - ps_t \qquad \text{where } ps_t = ps \quad \text{for } 1 \le t \le T$$

Provided that predictions of the future book return on owners' equity have been made, and that assessment of pr or ps can be made, (8a) or (8b) determines the expected growth of owners' equity in future periods.

2.3 Numerical examples based on residual income valuation

Finally in this section, some numerical results based on the valuation function (5) will be presented. The following assumptions have been made in this context:

- The horizon point in time t = T is either 5 or 10 years ahead. At this point in time, the relative measurement bias of owners' equity ($V_T/B_T - 1$) is expected to be 0, 0,5, 1,0, 1,5 or 2,0. The expected growth of owners'equity after the horizon point in time (δ') is set to 5%.

- The difference between the book return on owners'equity next year and the required rate of return (i.e. $R^*_{E,1} - \rho$) is –10%, 0, +10%, +20% or +30%.

  Furthermore, the book return increases/decreases linearly over time from $R^*_{E,1}$ to $R^*_{E,T+1}$.[9]

- The expected annual growth of owners' equity up to the horizon point in time t = T is 0, +5%, +10%, +15% or +20%.

- The required rate of return on owners' equity (ρ) is 10%.

[9] Values of $R^*_{E,2}, R^*_{E,3}, \ldots$ and $R^*_{E,T}$ have been calculated in accordance with the linear gradual change function in (7) above.

The ratio between the capital value and the book value of owners' equity at the valuation point in time, is presented in table 1 (the ratio is simply obtained by dividing the left- and right-hand side of (5) with B_0). The table shows that, for example, with T = 5, no accounting measurement bias at T = 5, an expected residual book return on owners' equity next year of +10% (implying that $R^*_{E,1}$ = 20%) and 10% annual growth of owners' equity up to t = T, the ratio between the capital value and the book value of owners' equity is 1,273. The capital value is hence 27,3% larger than the book value of owners' equity at the valuation point in time.

Horizon (T), Accounting bias (V_T/B_T - 1), and Growth (δ)	**Expected residual book return next period $(R^*_{E,1} - \rho)$:**				
	–10%	**0**	**+10%**	**+20%**	**+30%**
T = 5 and (V_T/B_T - 1) = 0					
Growth of owners' equity (δ):					
0	0,758	1,000	1,242	1,484	1,726
5%	0,743	1,000	1,257	1,514	1,770
10%	0,727	1,000	1,273	1,545	1,818
15%	0,710	1,000	1,290	1,580	1,869
20%	0,692	1,000	1,308	1,616	1,924
T = 5 and (V_T/B_T - 1) = 0,5					
Growth of owners' equity (δ):					
0	1,103	1,345	1,587	1,828	2,070
5%	1,179	1,436	1,693	1,949	2,206
10%	1,273	1,545	1,818	2,091	2,364
15%	1,387	1,676	1,966	2,256	2,546
20%	1,524	1,832	2,140	2,448	2,756
T = 5 and (V_T/B_T - 1) = 1,0					
Growth of owners' equity (δ):					
0	1,448	1,690	1,931	2,173	2,415
5%	1,615	1,872	2,128	2,385	2,642
10%	1,818	2,091	2,364	2,636	2,909
15%	2,063	2,353	2,643	2,933	3,222
20%	2,355	2,664	2,972	3,280	3,588
T = 5 and (V_T/B_T - 1) = 1,5					
Growth of owners' equity (δ):					
0	1,792	2,034	2,276	2,518	2,760
5%	2,051	2,307	2,564	2,821	3,078
10%	2,364	2,636	2,909	3,182	3,455
15%	2,739	3,029	3,319	3,609	3,899
20%	3,187	3,495	3,803	4,112	4,420
T = 5 and (V_T/B_T - 1) = 2,0					
Growth of owners' equity (δ):					
0	2,137	2,379	2,621	2,863	3,105
5%	2,486	2,743	3,000	3,257	3,514
10%	2,909	3,182	3,455	3,727	4,000
15%	3,416	3,706	3,996	4,285	4,575
20%	4,019	4,327	4,635	4,943	5,251

Table 1:
Capital value divided by book value of owners' equity (V_0/B_0), assuming a required rate of return ρ = 10%, a linear gradual change from $R^*_{E,1}$ to $R^*_{E,T+1}$, and an annual growth of owners' equity after the horizon point in time δ' = 5%.

Horizon (T), Accounting measurement bias (V_T/B_T - 1), and Growth (δ)	Expected residual book return next period ($R^*_{E,1} - \rho$): -10%	0	+10%	+20%	+30%
T = 10 and (V_T/B_T - 1) = 0					
Growth of owners' equity (δ):					
0	0,614	1,000	1,386	1,771	2,157
5%	0,562	1,000	1,438	1,875	2,313
10%	0,500	1,000	1,500	2,000	2,500
15%	0,425	1,000	1,575	2,150	2,724
20%	0,335	1,000	1,665	2,329	2,994
T = 10 and (V_T/B_T - 1) = 0,5					
Growth of owners' equity (δ):					
0	0,864	1,250	1,636	2,021	2,407
5%	0,953	1,391	1,828	2,266	2,704
10%	1,102	1,602	2,102	2,602	3,102
15%	1,341	1,916	2,491	3,066	3,640
20%	1,710	2,374	3,039	3,703	4,368
T = 10 and (V_T/B_T - 1) = 1,0					
Growth of owners' equity (δ):					
0	1,114	1,500	1,886	2,271	2,657
5%	1,344	1,781	2,219	2,656	3,094
10%	1,705	2,205	2,705	3,205	3,705
15%	2,257	2,832	3,407	3,982	4,556
20%	3,084	3,748	4,413	5,078	5,742
T = 10 and (V_T/B_T - 1) = 1,5					
Growth of owners' equity (δ):					
0	1,364	1,750	2,136	2,521	2,907
5%	1,734	2,172	2,609	3,047	3,485
10%	2,307	2,807	3,307	3,807	4,307
15%	3,173	3,748	4,323	4,898	5,472
20%	4,458	5,123	5,787	6,452	7,117
T = 10 and (V_T/B_T - 1) = 2,0					
Growth of owners' equity (δ):					
0	1,614	2,000	2,386	2,771	3,157
5%	2,125	2,562	3,000	3,438	3,875
10%	2,909	3,409	3,909	4,409	4,909
15%	4,089	4,664	5,239	5,814	6,389
20%	5,832	6,497	7,162	7,826	8,491

Table 1: (Continued)

Conditioned on the above assumptions, table 1 can be used as a guide of reference illustrating the relationship between V_0/B_0, the future growth of owners' equity and the future book return on owners' equity. Not surprisingly, the table shows that $V_0/B_0 = 1,0$ when the future book return on owners' equity is equal to the required rate of return and there is no accounting measurement bias at time $t = T$. A positive measurement bias results in higher 'value-to-book' ratios than when there is no measurement bias. The lowest value of V_0/B_0 (= 0,335) is observed for the combination of a negative residual book return next year (–10 %), the horizon point in time 10 years ahead, no accounting measurement bias at $t = T$ and an expected high future growth of owners' equity (+20%). Obviously, a situation of this kind is somewhat unrealistic – a persistent negative residual return on owners' equity is unlikely to go along with such a high growth rate. (With an expected growth rate δ between 0 and +5%, V_0/B_0 would – ceteris paribus – be about 0,6.)

Companies with positive residual book returns in the future are associated with values of V_0/B_0 larger than 1,0 in the table. In the absence of any accounting measurement bias, the highest 'value-to-book' is 2,994, pertaining to a company with an expected residual book return on owners' equity of +30% next year, the horizon point in time 10 years ahead and an annual growth rate of 20% up to this point in time. Higher 'value-to-book' ratios are obtained for companies with a positive measurement bias at the horizon point in time. If, e.g. $(V_T/B_T - 1) = 1,0$ or $2,0$ – without changing any other assumptions – V_0/B_0 would be about 5,7 or 8,5. Evidently, the ratio of capital value to book value of owners' equity can be very high for profitable and growing companies with substantial self-generated immaterial assets (i.e. typically uncapitalized research and development expenses and/or uncapitalized expenditures for marketing or personnel training).

3. Value Added Valuation

3.1 Model Specification

Cash flow valuation commonly includes two main steps – the valuation of invested capital in the company (= the sum of owners' equity and financial net debt) and the valuation of company financial net debt. The value of invested capital is determined as the present value of expected future 'free cash flows' = 'cash flows to investors'. The value of owners' equity is then calculated as the difference between the value of the invested capital and the financial net debt. The modelling logic hence implies:

(9) $$V_0 = V(ONA_0) - V(ND_0)$$

where: $V(ONA_0)$ = value of invested capital in the company at time t = 0

ONA_0 = operating net assets at time t = 0
= operating assets less operating liabilities at time t =0
= book value of owners' equity plus financial net debt at time t = 0

$V(ND_0)$ = value of company financial net debt at time t = 0

ND_0 = company financial debt less financial assets (= net debt) at time t = 0

With regard to the relationship in (9), note that 'invested capital in the company' coincides with the difference between operating assets and operating liabilities (= operating net assets). Also, note that the operating assets typically include some financial assets necessary for the operations of the company. (The financial net debt is consequently measured net of financial assets which are viewed as 'unnecessary' for the operating business activities of the company.)

A kind of 'standard procedure' nowadays when determining $V(ONA_0)$ in (9) is to calculate the present value of expected future 'free cash flows', generated by the operating net assets of the company:[10]

$$(10) \qquad V(ONA_0) = \sum_{t=1}^{T} \frac{FCF_t}{(1+r_{wacc})^t} + \frac{V(ONA_T)}{(1+r_{wacc})^T}$$

where: FCF_t = expected free cash flow (after company income taxes) generated by the company during period t

$V(ONA_T)$ = expected value of invested capital in the company at the horizon point in time t = T

r_{wacc} = weighted average cost of capital (after company income taxes)

The measure of free cash flow in (10) is reduced by an 'adjusted' income tax cost, i.e. the income tax that the company would have paid if the operating net assets had been financed by owners' equity only. The weighted average cost of capital is measured according to the well-known formula:

$$(11) \qquad r_{wacc} = \rho \cdot (1 - L) + r_D \cdot (1 - T_c) \cdot L$$

where: L = company target leverage ratio $V(ND_t)/V(ONA_t)$

r_D = required rate of return on company financial net debt

T_c = company tax rate

The value generating attribute in (10) is the 'free cash flow' that would have been available to the shareholders of the company if the operating net assets had been financed by owners' equity only. Strictly speaking, the valuation function is based on the idea that equity investors directly have access to the cash flows being generated by the

[10] The valuation approach is often referred to as 'free cash flow valuation' or, with reference to Copeland, Koller & Murrin (2000), 'McKinsey valuation'. However, the valuation technique as such has been known in the finance literature since the 1960's.

operating activities of the company. (This can certainly be the case when a company has – or will have after a takeover has taken place – one dominating owner, who directly has access to the company free cash flows.)

If the accounting for operating net assets follows the 'operating assets relation' (a relationship corresponding to the 'clean surplus relation' specified in sub-section 2.1 above)[11], we have:

(12) $$ONA_t = ONA_{t-1} + EBIT_t(1 - T_c) - FCF_t$$

where: $EBIT_t$ = operating earnings (before net interest expense and taxes) accrued in period t

Rewriting (12) and introducing a measure of return on operating net assets (after company income taxes), $R^*_{ONA,t} \equiv EBIT_t(1 - T_c)/ONA_{t-1}$, the measure of free cash flow in (10) can be expressed as:

(13) $$FCF_t = ONA_{t-1} \cdot R^*_{ONA,t} - (ONA_t - ONA_{t-1})$$

Viewing r_{wacc} as the 'normal' return on operating net assets, $R^*_{ONA,t}$ can trivially be written as $r_{wacc} + (R^*_{ONA,t} - r_{wacc})$ and (13) can be reexpressed as follows:

(13') $$FCF_t = ONA_{t-1} \cdot [r_{wacc} + (R^*_{ONA,t} - r_{wacc})] - (ONA_t - ONA_{t-1})$$

Combining (13') with the free cash flow valuation function in (10), a new valuation specification – a 'value added' valuation model – can be derived:

[11] Cf. Feltham & Ohlson (1995).

(14) $$V(ONA_0) = ONA_0 + \sum_{t=1}^{T} \frac{ONA_{t-1}(R^*_{ONA,t} - r_{wacc})}{(1+r_{wacc})^t} + $$

$$+ \frac{ONA_T(V(ONA_T)/ONA_T - 1)}{(1+r_{wacc})^T}$$

Similarly to the 'residual income' valuation model (cf. (4) on p. 4 above), (14) shows that the value of the invested capital in a company can be calculated as the sum of the following components:

- Accounting book value – the book value of operating net assets (= owners' equity + net debt) at the valuation point in time.
- Present value of the expected abnormal operating profitability ('value added ') of the company until the horizon point in time.
- Present value of the expected goodwill/badwill of operating net assets at the horizon point in time (i.e. $ONA_T(V(ONA_T)/ONA_T - 1) = V(ONA_T) - ONA_T$).

Note that (14) is deduced from the free cash flow valuation function in (10), implying that a calculated value of invested capital according to (14) will coincide with a value calculated in accordance with (10) (as long as the 'operating assets relation' holds). Even if no measure of free cash flow is explicitly recognized in (14), this measure still constitutes the underlying value generating attribute.

3.2 Assessments of valuation parameters

In order for (14) to be applicable, predictions of ONA_{t-1}, $R^*_{ONA,t}$ and $(V(ONA_T)/ONA_T - 1)$ have to be made. In this regard, the methodology discussed in connection with the 'residual income' valuation model in sub-section 2.2 above, can be suggested. The methodology includes – in short – the following steps:

1. A value of the relative accounting measurement bias ($V(ONA_T)/ONA_T - 1$) at the horizon point in time t = T (when the business goodwill of the company is expected to be negligible) is assessed. An expected value of $R^*_{ONA,T+1}$ being consistent with this accounting measurement bias, can then be calculated as follows:[12]

$$(15) \quad R^*_{ONA,T+1} = r_{wacc} + (V(ONA_T)/ONA_T - 1)(r_{wacc} - \delta'_{ONA})$$

where: δ'_{ONA} = expected annual growth of operating net assets after time t = T

Note that (15), in principle, corresponds to (6) in sub-section 2.2 of the paper.

2.a) The expected return on operating net assets for next year, $R^*_{ONA,1}$, is predicted.

b) The expected time series adjustment from $R^*_{ONA,1}$ to $R^*_{ONA,T+1}$ is assessed and expected values of $R^*_{ONA,t}$ for t = 2, t = 3,... to t = T are predicted.

3. Expected values of the growth rate of operating net assets for years t = 1, t = 2, ... to t = T are predicted, whereby expected values of ONA_1, ONA_2,... to ONA_T can be assessed. (In forecasting the growth of operating net assets, it is often helpful to start out with predictions of the expected company sales growth.)[13]

Armed with the 'value added' valuation model in (14) and the above assessments, a value of the invested capital of the company can be calculated. Subtracting the value of the financial net debt of the company, the value of owners' equity is obtained (in accordance with (9) above). The next valuation problem is hence to determine the value of the financial net debt.

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[12] Assuming that the assessed value of ($V(ONA_T)/ONA_T - 1$) holds for all periods after the horizon point in time and the expected growth in operating net assets (= δ'_{ONA}) is a constant, expression (15) will hold for all periods T+1, T+2,…T+ ∞.

[13] A thorough discussion of the prediction problems in question can be found in Copeland, Koller & Murrin (2000), pp. 233-266, and in Jennergren (1998) pp. 8-22.

The accounting book value of the financial net debt at t = 0 is a 'naive' benchmark estimate of the capital value of the financial net debt. Whether this is a reasonable estimate or not, depends on the accounting principles being used by the company – if financial assets and liabilities are accounted for according to 'mark-to-market' accounting, the approach is hardly troublesome. If, on the other hand, the principles of prudence and realization affect the accounting book values, the approach can be more or less misleading.

If the financial assets and liabilities of the company are associated with the same investment risk, as being reflected by the required rate of return on company debt (r_D), a valuation function of financial net debt can actually be deduced in conformity with the 'value added' valuation model:

$$(16) \qquad V(ND_0) = ND_0 + \sum_{t=1}^{T} \frac{ND_{t-1}(R_{ND,t} - r_D)}{(1+r_D)^t} + \frac{ND_T(V(ND_T)/ND_T - 1)}{(1+r_D)^T}$$

where: $R_{ND,t}$ = net interest cost accrued in period t

The net interest cost in (16) is calculated as the difference between financial expenses (Fe_t) and financial revenues (Fr_t), divided by the financial net debt at the beginning of the period (i.e. $R_{ND,t} \equiv (Fe_t - Fr_t)/ND_{t-1}$).

The valuation function in (16) clearly shows that $V(ND_0) = ND_0$ if $R_{ND,t} = r_D$ and $(V(ND_T)/ND_T - 1) = 0$, which in principle would be the case if 'mark-to-market' and 'clean surplus' accounting is used. When the accounting principles are conservatively biased, $V(ND_0)$ will be equal to ND_0 if the financial net debt has a floating interest rate (whereby $R_{ND,t} = r_D$ and $ND_t = V(ND_t)$). Also, $V(ND_0)$ will be approximately equal to ND_0 in situations when the financial net debt is approaching its maturity date, or if it originates from historical periods when the borrowing and lending rates were about the same as the required rate of return on company net debt at time t = 0.[14] In other situations, (16) is rather to be recommended for the valuation of the financial net debt. The general methodology being discussed above – in the context of the valuation of

[14] Cf. the discussion in White, Sondhi & Fried (1997), pp. 497-500.

operating net assets – would then be suggested also for the prediction of future values of $R_{ND,t}$, ND_{t-1} and $(V(ND_t)/ND_t – 1)$.

4. On the 'usefulness' of the valuation models

The valuation models which have been specified in section 2 (residual income valuation) and section 3 (value added valuation) of the paper, are based on the following generic logic of financial analysis:

(18) Capital value = (Book value of capital) +
+ (Present value of future abnormal earnings) +
+ (Present value of goodwill/badwill at the horizon point in time)

In the 'residual income' valuation model, the 'book value of capital' is the book value of owners' equity and the measure of 'abnormal earnings' is based on the accounting net income. In the 'value added' valuation model, the 'book value of capital' is equal to the book value of owners' equity plus the financial net debt, i.e. the book value of operating net assets, and the measure of 'abnormal earnings' is based on operating earnings.

What are then the potential advantages – or disadvantages – of the valuation models which have been outlined above? With regard to the 'residual income' valuation model, it can first be noted that the underlying value generating attribute is a measure of (net-)dividends. In principle, this implies that the expected future (net-)dividends of the company have to be predicted. However, provided that the clean surplus relation of accounting holds, the prediction of (net-)dividends can be replaced by predictions of the book return on owners' equity ($R^*_{E,t}$), future book values of owners' equity (B_{t-1}) and an estimate of the relative measurement bias of owners' equity at the horizon point in time (V_T/B_T - 1). This can simplify the prediction problem at hand, especially when the equilibrium relationship between $R^*_{E,T+1}$, (V_T/B_T - 1) and the required rate of return ρ – as expressed in (6) above – is recognized.

In the specification of the residual income valuation model in subsection 2.1 of the paper, it has been assumed that the required rate of return is a constant. In situations when the capital structure of the company is expected to remain unchanged over time, this is normally not a problematic assumption. If the capital structure of the company is expected to change over time, however, the required rate of return should typically not be treated as a constant. Disregarding costs of financial distress[15] and assuming that the personal tax rate on dividends and interest income are the same, the following relationship then holds:[16]

$$(19) \qquad \rho = \rho_u + (\rho_u - r_D)\frac{(1 - T_c) \cdot V(ND_t)}{V_t}$$

where: ρ_u = required rate of return on owners' equity for a company financed by owners' equity only

(19) shows that the required rate of return on owners' equity is a positive function of the debt-to-equity ratio $V(ND_t)/V_t$ and that ρ increases by $(\rho_u - r_D)(1 - T_c)$ when this ratio increases by one unit. If eg. $\rho_u = 8\%$, $r_D = 5\%$ and $T_c = 0{,}30$, ρ increases from 12,2% to 14,3% if the debt-to-equity ratio goes from 2,0 to 3,0. If changes in the future leverage of the company are disregarded in the 'residual income' valuation model, this indicates that a calculated value of owners' equity can be somewhat erroneous.

It is certainly possible to handle expected future changes in the debt-to-equity ratio, letting the required rate of return on owners' equity change over time in accordance with (19). A modification of the valuation model of this kind is not trivial, however, as future periodic values of ρ must be determined based on the debt-to-equity ratio at the beginning of future periods. These values have to be consistent with V_0, as well as with the implied values V_1, V_2,... to V_T. In general, this calls for an iterative (computorized) dynamic solution procedure.

In the 'value added' valuation model, the underlying value generating attribute is a measure of company free cash flow. Since free cash flows typically fluctuate strongly over time, trying to forecast expected values of future free cash flows can be (more or

[15] The importance of costs of financial distress is discussed in e.g. Brealey & Myers (2003), pp. 497-510.

[16] See e.g. Copeland & Weston (1988), chapter 13.

less) futile. In the 'value added' valuation model, however, the after tax return on operating net assets ($R^*_{ONA,t}$), book values of operating net assets (ONA_{t-1}) and an assessment of the relative accounting measurement bias at the horizon point in time ($V(ONA_T)/ONA_T - 1$), replace the free cash flows. In general, it can be expected to be less difficult to forecast these measures, than having to predict expected values of free cash flows per se. Especially the equilibrium relationship between $R^*_{ONA,T+1}$, ($V(ONA_T)/ONA_T - 1$) and the required rate of return r_{wacc} – as expressed in (15) above – is likely to be helpful in this context.

The weighted average cost of capital r_{wacc} is assumed to be constant over time in the 'value added' valuation model as it has been specified in subsection 3.1. In accordance with the discussion above of the required rate of return on owners' equity in the 'residual income' valuation model, a constant r_{wacc} might be problematic in situations when future changes in the capital structure are to be expected. Disregarding costs of financial distress and assuming that the personal tax rate on dividends and interest income are the same, the following relationship between r_{wacc} and the leverage ratio can be specified:[17]

(20) $$r_{wacc} = \rho_u[1 - T_c \cdot (V(ND_t)/V(ONA_t))] = \rho_u[1 - T_c \cdot (1 - V_t / V(ONA_t))] = \\ = \rho_u[1 - T_c \cdot (1 - 1/(1 + V(ND_t)/V_t)]$$

(20) shows that r_{wacc} decreases as the debt-to-equity ratio $V(ND_t)/V_t$ increases. The function is non-linear – the reduction of r_{wacc} successively diminishes as the value of the ratio increases. In this respect, there is a difference as compared to the sensitivity of the required rate of return on owners' equity ρ to variations in the debt-to-equity ratio. Especially when the value of $V(ND_t)/V_t$ is high, the sensitivity of r_{wacc} can be low. If e.g. $V(ND_t)/V_t$ increases from 2,0 to 3,0 when ρ_u = 8%, r_D = 5% and T_c = 0,30, r_{wacc} decreases by 0,0020 (from 6,40% to 6,20%). If, on the other hand, the leverage ratio increases from 6,0 to 7,0 , r_{wacc} is reduced by only 0,0004 (from 5,94% to 5,90%) with the same set of assumptions. Furthermore, r_{wacc} is less sensitive to changes in the debt-to-equity ratio when the company tax rate is low; r_{wacc} is *de facto* equal to ρ_u for all

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[17] See e.g. Copeland & Weston (1988), pp. 451-457.

values of $V(ND_t)/V_t$ if $T_c = 0$. This really indicates that the assumption of a constant r_{wacc} in the 'value added' valuation model is less problematic than the assumption of a constant ρ in the 'residual income' valuation model.

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