Terminal Value Techniques in Equity Valuation
- Implications of the Steady State Assumption ♦

Joakim Levin ♠

Per Olsson ♣

SSE/EFI Working Paper Series in Business Administration No 2000:7

June 2000

Abstract: This paper examines the conditions necessary for calculating steady state terminal values in equity (company) valuation models. We make explicit use of the fact that a company's income statements and balance sheets can be modeled as a system of difference equations. From these difference equations, we derive conditions for steady state. The conditions ensure that the company remains qualitatively similar year by year after the valuation horizon and that it has a stable development of earnings, free cash flows, dividends and residual income. We show how steady state condition violations cause internal inconsistencies in valuation models and how this can have a substantial impact on the value estimates. Steady state is further a necessary condition for a free cash flow valuation, a dividend valuation and a residual income valuation to yield identical results when terminal values are used. The parameters of the model are common accounting and control concepts, and the derived conditions have accounting meaning, linking stock variables in the balance sheet with the flow variables in (and related to) the income statement.

Key words: equity valuation, terminal value, financial statement analysis

JEL codes: G12, G31, M49

♦ This paper has previously been circulated under the title *Horizon Values in Company Valuation*. The helpful comments of Peter Easton, Jennifer Francis, Frøystein Gjesdal, Peter Jennergren, Thore Johnsen, Kenth Skogsvik, Stefan Yard, and two anonymous referees as well as workshop participants at the EURO Working Group on Financial Modelling, the Norwegian School of Economics and Business Administration, the Ohio State University, and the Stockholm School of Economics are gratefully acknowledged. Financial support from the *Economic Research Institute* and the *Bank Research Institute*, Stockholm, Sweden, is gratefully acknowledged.

♠ Joakim Levin (cjl@hhs.se), Stockholm School of Economics, P.O.Box 6501, SE-11383 Stockholm

♣ Corresponding author – Per Olsson (polsson@bus.wisc.edu), University of Wisconsin-Madison, 975 University Avenue, Madison, Wisconsin 53706

1 Introduction

This paper explores several issues associated with the calculation of continuing value formulas used as horizon values (or terminal values) in company and equity valuation models. Our primary focus is on clarifying the steady state assumption that underlie the use of such horizon values, and the conditions necessary to make this assumption operational. The derived conditions have intuitive accounting meaning and are shown to guarantee internal consistency in valuation models. The conditions furthermore guarantee that popular equity valuation models derived from the PVED principle (present value of expected dividends), such as the Residual Income model, the Free Cash Flow model and the Dividend Discount model, yield identical results also when using horizon values. These models are theoretically equivalent, and yet large-sample comparisons of the valuation models (Penman and Sougiannis [1998] and Francis, Olsson and Oswald [2000]) show that the models yield vastly different results, even when they are based on the same set of pro forma financial statements and all assumptions are identical. We show how this anomaly can be caused by steady state condition violations.

In equity valuation models based on forecasted financial statement data (see, e.g., Palepu, Bernard and Healy [1996] or Copeland, Koller and Murrin [1994]), one commonly forecasts financial statements for a finite number of years – the explicit forecast period. For each year a valuation attribute (e.g., free cash flow or residual income) is calculated from the financial statement forecasts. The horizon value refers to the present value (at the horizon) of the valuation attribute after the explicit forecast period.

Generally, this valuation approach can be formulated as:

(1) $$V_t = \sum_{s=t+1}^{H} \frac{VA_s}{(1+k)^{s-t}} + \frac{PV_H^{VA}}{(1+k)^{H-t}}$$

where:

where:	V_t	is the estimated value (of a particular valuation concept) at the valuation date t
	H	is the horizon (the last year of the explicit forecast period)
	VA	is the valuation attribute
	PV_H^{VA}	is the horizon value
	k	is the discount rate

Depending on the valuation attribute, formula (1) may need a correction to yield the equity value. So, for example, will net debt and preferred stock be deducted in the Free Cash Flow model, whereas book value of stockholders' equity will be added in the Residual Income model.

We will consider four commonly used valuation attributes: earnings, free cash flows, dividends and residual income. Free cash flows, dividends and residual income can be directly used in valuation models that are formally equivalent to the principle that equity value equals the present value of all future expected dividends. Earnings are often used in less elaborate valuation models, sometimes with additional assumptions (e.g., an assumed dividend payout ratio).

As Brealey and Myers [1991, p.64] suggest, the rationale for using a horizon value is pragmatic: "Of course, the [...] business will continue after the horizon, but it's not practical to forecast free cash flow year by year to infinity." The most common method of calculating horizon values is to use a continuing value formula:

(2) $$PV_H^{VA} = \frac{VA_{H+1}}{k-g}$$

where: PV_H^{VA} is the relevant horizon value

VA is the valuation attribute

k is the discount rate

g is the growth rate ($g < k$)

To calculate (1), one needs a forecast of VA_{H+1}; this is often achieved by letting the valuation attribute at time *H* grow by *g*, the growth rate:

$$(3) \qquad PV_H^{VA} = \frac{(1+g)VA_H}{k-g}$$

When using an expression such as (2) or (3) to calculate horizon values, it is assumed that the valuation attribute grows at a constant rate and that the discount rate remains constant.[1]

The fact that the horizon value is often calculated using a very simple formula does not indicate that it is unimportant. On the contrary, Copeland, Koller, and Murrin [1994, p. 275] report typical values for some industries: for a company in the tobacco industry the horizon value accounts for 56% of the total company value, in the sporting goods industry it is 81%, for the typical skin care business the figure is 100% and for a high tech company 125% (the figures are calculated using a horizon eight years into the future for the Free Cash Flow model).[2]

The valuation attribute derived from the financial statements for year *H* is commonly used as numerator in the continuing value (as in expression (3)), sometimes with some adjustments to 'normalize' the valuation attribute to a level that is deemed sustainable in the post-horizon period, possibly with growth. The level of theoretical justification for this varies, but a general theme is a reference to the steady state concept in which the company remains qualitatively similar year by year after the horizon. Kaplan and Ruback [1995, p. 1064], e.g., place a restriction on the 'terminal capital cash flow' (their valuation attribute at the horizon) by setting depreciation and amortization equal to capital expenditures, noting that depreciation and amortization cannot exceed capital expenditures in steady state. The latter restriction visualizes, but does not make explicit, that there is a link between the perceived reasonability of the valuation attribute at the horizon and the underlying fundamentals as expressed in forecasted financial statements.

Adjustments made (or not made) to the valuation attribute used in the continuing value calculation can have a large impact on the entire valuation, so it is important *how* these adjustments are made. One can come up with intuitive restrictions, for example the above-mentioned condition that depreciation and amortization not exceed capital expenditures in steady state; however, such intuitive restrictions may be ad-hoc and incomplete. Our objective in this study is to suggest a more systematic approach, making explicit use of properties of the accounting system. In particular we note that the time series of forecasted financial statements can be seen as a system of difference equations. Seen in that light the steady state concept can be made operational by mathematical analysis, where all conditions necessary for steady state will be derived as initial value conditions on the system of difference equations. While the algebra for showing this may at times be tedious, the result turns out to be easy to implement and, we think, intuitively appealing.

The intuition is as follows: When a firm enters into steady state its qualitative behavior is expected to remain the same year after year. Qualitative behavior can be made operational by decomposing it into common accounting and control concepts, such as profit margin, sales growth, productivity ratios relating sales to capital, etc. The constancy of these parameters over time is a necessary condition for steady state to prevail; it is not, however, sufficient. One also has to consider the interaction of these parameters, noting that their values cannot be set independently. Furthermore, parameter values must be set such that stock relates to flow in a reasonable manner in the forecasted financial statements. All in all, these interdependencies quickly become complicated, and our modeling can be seen as a way of dealing with the complexity. Our analysis shows the relations among parameters and the associations with stock variables that must hold for steady state.

Steady state conditions matter in a technical sense because they ensure that all implicit assumptions behind continuing values are fulfilled. More fundamentally, steady state conditions ensure that the forecasted performance is stable. If steady state conditions are violated, the qualitative behavior of the company will change after the valuation horizon, and will keep changing over time. Such changes contradict the premise for using a valuation horizon – namely that the company is expected to be stable at the horizon, and hence generate stable earnings, cash flows, etc. As we demonstrate, even minor internal inconsistencies can have a substantial impact on the final value estimate of a company. The steady state conditions can also be useful in the process of determining at which point in time the horizon itself should be set.

The main result is of a normative nature: all flows (income statement and related variables) in the first year after the horizon should be decided such that corresponding stocks (balance sheet variables) grow at the revenue growth rate. This rule ensures that the company remains qualitatively similar throughout the post-horizon period. This is also a necessary condition for terminal value calculations in the Free Cash Flow model, the Dividend Discount model and the Residual Income model to yield identical results. That these theoretically equivalent valuation models can yield different results even when they are based on the same set of forecasted financial statements and all assumptions are identical is a source of confusion both in the literature and in the classroom. We show how steady state condition violations can be the cause of such differences.

Our analysis provides a methodology that is quite general and can be applied to any valuation framework that involves forecasts of future balance sheet and income statement data. We do not claim that all company valuation models *must* include steady state horizon values. We do claim, however, that the vast majority of valuation texts and applications include this type of horizon values and that it is therefore of great practical importance to have the steady state issue thoroughly investigated and implementation routines for steady state developed.

The rest of the paper is organized as follows. In the next section, we describe our methodology and selected modeling choices and define the steady state concepts associated with each of the four valuation attributes we consider. Section 3 derives the conditions necessary to achieve steady state for each attribute and for the entire balance sheet. Section 4 argues the empirical importance of the results, exemplifies by a case study and discusses some related large sample findings. Section 5 explains the transition from the explicit forecast period to steady state and further implementation issues. Section 6 summarizes and concludes.

2 Terminology and Definitions

We set up a valuation model for the (steady state) period, where the company's qualitative behavior is expected to remain the same year after year. We define the company’s balance sheet and income statement as follows:

Assets	**Debt and Equity**
Net Working Capital	Debt
Net Property, Plant and Equipment	Deferred Taxes
= Gross PPE – Accumulated Depr.	Equity

Revenues
-Operating Expenses
-Depreciation Expense
-Interest expense
-Taxes
Earnings

We further define the following parameters. Many of them are ratios, and the term ratio analysis is sometimes used also to describe this kind of valuation:[3]

a net working capital as % of revenues (sales)

b gross PPE as % of revenues (sales)

c increase in deferred taxes as % of gross PPE

d depreciation expense as % of preceding year's gross PPE

g nominal growth rate, revenues (sales)

i interest rate on debt

k discount rate [4]

p operating expenses as % of revenues (sales)

r retirements as % of preceding year's gross PPE

τ tax rate

w debt as % of balance sheet total (book value)

The following state-variables are also defined:

R_t revenues (sales) in year t,

A_t accumulated depreciation at the end of year *t*,

T_t deferred taxes at the end of year *t*.

We can now express the balance sheet and income statement as functions of the state variables:

Assets	
Net Working Capital	aR_t
Net Property, Plant and Equipment = Gross PPE – Accumulated Depr.	 $bR_t - A_t$ where $A_t = A_{t-1} + (d - r)bR_{t-1}$

Debt and Equity	
Debt	$w(aR_t + bR_t - A_t)$
Deferred Taxes	$T_{t-1} + cbR_t$
Equity	$(1-w)(aR_t + bR_t - A_t) - T_t$

Revenues	R_t
-Operating Expenses	$-pR_t$
-Depreciation Expense	$-dbR_{t-1}$
-Interest Expense [5]	$-iw(aR_{t-1} + bR_{t-1} - A_{t-1})$
-Taxes [6]	$-\tau(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$
Earnings	$(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$

Some modeling choices are straightforward, such as defining operating expense as a percentage of revenues. Others are more debatable – in particular the determination of PPE-related items, including depreciation and retirements, is not self-evident. The methodology in this paper lends itself to any specification, but to exemplify we have chosen a specification intended to resemble the verbal exposition in Copeland, Koller and Murrin [1990, 1994]. This has the advantage of directly coupling our analytical results to a valuation book well known among practitioners, students and academics.

Our use of a single parameter to determine net working capital implies that all working capital items can be defined as a fraction of revenues and, therefore, that they in steady state can be aggregated into *net* working capital, governed by the same parameter. Gross PPE is also determined as a percentage of revenues. It is thus assumed that it takes a stable amount of working capital and physical assets to generate each dollar of sales. Accumulated depreciation is the prior period's accumulated depreciation plus the current period's depreciation expense minus the book value of assets retired in the current period.

Book value of debt is defined as a percentage of the balance sheet through the parameter *w*, implying that the company tries to maintain a constant debt-value ratio in book value terms.[7] A target leverage is a common assumption in valuation texts. Empirically, Fama and French [1997] also show that there is a slow mean reversion to (firm-specific) target leverage. Hence, it seems reasonable to model leverage as constant in the steady state period. We begin by modeling this assumption in book-value terms, and later derive the additional conditions for it to hold in market value terms. Deferred taxes are modeled as a separate debt item.[8] Book value of equity is the residual item of the balance sheet.

To further link the income statement with the balance sheet over time, we assume the clean surplus relation holds. This means that the change in book value of equity equals earnings minus dividends, where dividends are defined net of capital contributions/withdrawals. As mentioned above, book value of equity is the residual item of the balance sheet. Through the clean surplus relation dividends are defined to be the residual item of the entire equations system, with any excess capital distributed to equity owners. For simplicity, the book value of debt is assumed to equal its market value. It should also be noted that there are no excess marketable securities; we define debt as a net financial item. Cash needed to support operations is included in net working capital, however.

The following valuation attributes can now be derived:

Earnings, X_t:

(4) $$(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))$$

Free cash flow, FCF_t: [9]

(5) $$\begin{aligned}&(1-\tau)(R_t - pR_t - dbR_{t-1}) + dbR_{t-1} + (T_t - T_{t-1})\\ &-(aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1})\end{aligned}$$

Dividends, DIV_t:

(6) $$\begin{aligned}&(1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) - T_{t-1}\\ &+(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))\\ &-((1-w)(aR_t + bR_t - A_t) - T_t)\end{aligned}$$

Residual income, RI_t: [10]

(7) $$\begin{aligned}&(1-\tau)(R_t - pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1}))\\ &-\ k_E((1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) - T_{t-1})\end{aligned}$$

We distinguish among the following types of steady state:

Parametric steady state (PSS) means that the parameters describing the company's development are constant, e.g., a constant revenue growth, a constant profit margin, etc. PSS is the weakest form of steady state, because no restrictions are placed on the parameters other than that they remain constant.

Earnings steady state (ESS) means that the company's predicted earnings will grow at a constant rate. More precisely, earnings in any year t+1 are described by $X_{t+1} = (1+g)X_t$, where g is the constant revenue growth rate.

Free cash flow steady state (FSS) means that the company's predicted free cash flow will grow at a constant rate g: $FCF_{t+1} = (1+g)FCF_t$.

Dividend steady state (DSS) means that the company's predicted dividends will grow at a constant rate *g*: $DIV_{t+1} = (1+g)DIV_t$.

Residual income steady state (RSS) means that the company's predicted residual income will grow at a constant rate *g*: $RI_{t+1} = (1+g)RI_t$.

Balance sheet steady state (BSS), finally, means that all items on the company's balance sheet will grow at a constant rate *g*.

To obtain steady state in the valuation attributes (ESS, FSS, DSS, RSS), it is not, mathematically speaking, a necessary condition that the company is in parametric steady state. The development of the underlying parameters may be shifting over time but in an offsetting way so that the valuation attribute still grows at a constant rate. We have never seen such a model proposed neither in textbooks nor in the research literature, however, so we will abstract from that possibility and treat parametric steady state as a necessary condition. The intuitive interpretation of steady state as a company that remains qualitatively similar year by year lends support, we think, to this choice.

The nature of the growth rate also deserves further comment. Using a continuing value implies an infinite time series after the horizon. Hence it must really be the same growth rate in different variables and attributes, and this growth rate must be the revenue growth rate. If, e.g., earnings were to have a higher growth rate than revenues, then earnings would after a few years exceed revenues. While this sounds like a fantastic business, it is hardly realistic. Reversed, if earnings grow slower than revenues, then profitability will approach zero – also not a realistic steady state scenario for a going concern, since any rational owner would terminate operations and take the funds elsewhere. A similar logic applies to the other attributes as well.[11]

3 Steady State Development

3.1 Introduction

The company is expected to have reached a point in time where the parameters governing the company's balance sheets and income statements are constant, i.e. the parameters have reached their steady state values and the company is in parametric steady state (PSS). In this section, we attempt to investigate under which conditions on the input parameters PSS will imply steady state also for the valuation attributes (ESS, DSS, FSS, RSS).

In Section 2, three state variables were defined forming the following system of difference equations:

(8) $R_t = (1+g)R_{t-1}$ (revenues in year *t*)

(9) $A_t = A_{t-1} + (d-r)bR_{t-1}$ (accumulated depreciation at the end of year *t*)

(10) $T_t = T_{t-1} + cbR_t$ (deferred taxes at the end of year *t*)

By solving the system, analytical expressions for the state variables can be obtained; from these, expressions for the valuation attributes will be derived. The solution to the system of difference equations is (see Appendix 1 for details):

(11) $R_t = (1+g)^t R_0$

(12) $$A_t = A_0 + \frac{(1+g)^t - 1}{g}(d-r)bR_0$$

(13) $T_t = T_0 + \left[\frac{(1+g)^{t+1} - 1}{g} - 1 \right] cbR_0$

R_0, A_0 and T_0 are the initial values of revenues, accumulated depreciation and deferred taxes, respectively. These initial values for the post-horizon period are given by the forecasts of complete financial statements made for the explicit forecast period.[12]

3.2 Steady state in valuation attributes

Earnings steady state (ESS)

Earnings, X_t, are given by expression (4). Rearranging and substituting expressions (11) and (12) into (4) yields:

(14) $$X_t = (1+g)^t R_0 (m - z_X) - \chi(\gamma R_0 - A_0) \qquad \text{for } t \geq 1$$

with the following constants:

$$m = (1-\tau)(1-p)\ , \quad z_X = \frac{(1-\tau)\left(db + iw\left(a + b - \frac{d-r}{g}b\right)\right)}{1+g}, \quad \chi = (1-\tau)iw\ , \quad \gamma = \frac{(d-r)}{g}b$$

Expression (14) says that for ESS to hold, earnings must grow by g. The existence of the second term in (14) implies that earnings will not grow at the constant rate g unless the second term is zero:

(15) $$\chi(\gamma R_0 - A_0) = 0$$

Or in terms of the original parameters:

(15a) $$(1-\tau)iw\left(\frac{d-r}{g}bR_0 - A_0\right) = 0$$

Two possibilities for condition (15a) to hold are ruled out by assumption: i=0 (zero interest rate) and τ=1 (100% tax rate). This leaves two possible conditions, one of which must hold for earnings to grow at the rate g: (i) all-equity financing (w=0); or (ii) $gA_0 = (d-r)bR_0$. The left-hand side of condition (ii) is the growth in accumulated depreciation in the first year of the steady state. The right-hand side is depreciation expense minus retirements in the first year of the steady state period. Intuitively, this condition establishes the required link between accumulated depreciation (the stock variable) and depreciation-retirements (the flows that affect that stock variable). We explain and elaborate on these interpretations at the end of this section.

Free cash flow steady state (FSS)

By substituting expressions (8), (10) and (11) into (5), the following expression is obtained:

(16) $$FCF_t = (1+g)^t R_0 (m - z_{FCF}) \qquad \text{for } t \geq 1$$

with the following constants:

$$m = (1-\tau)(1-p)\ , \qquad z_{FCF} = \frac{-\tau db - (1+g)cb + ga + (g+r)b}{1+g}$$

Expression (16) tells us that free cash flow steady state is an immediate consequence of the constancy of the input parameters; no further parameter restrictions are required. Compared to earnings, free cash flow is thus considerably less restricted.[13]

Dividend steady state (DSS)

Rearranging expression (6), and substituting expressions (10 - 12) yields:

(17) $$DIV_t = (1+g)^t R_0 (m - z_{DIV}) - \chi(\gamma R_0 - A_0) \qquad \text{for } t \geq 1$$

with the following constants:

$$m = (1-\tau)(1-p), \quad \chi = (1-\tau)iw\ , \quad \gamma = \frac{(d-r)}{g}b$$

$$z_{DIV} = \frac{-\tau db - (1+g)cb + ga + (g+r)b - gw\left(a + b - \frac{d-r}{g}b\right) + (1-\tau)iw\left(a + b - \frac{d-r}{g}b\right)}{1+g}$$

The dividends expression (17) is similar to the earnings expression (14). This is not surprising, since dividends are defined through the clean surplus relation. Although *z* differs for dividends vs. earnings, that difference is irrelevant for purposes of establishing steady state; for dividend steady state to hold the same conditions as for earnings steady state apply.

Residual income steady state (RSS)

Substituting expressions (11 - 13) into expression (7) yields the expression for residual income:

(18) $$RI_t = (1+g)^t R_0 (m - z_{RI}) - (\chi + \vartheta)(\gamma R_0 - A_0) - k_E(\kappa R_0 - T_0) \qquad \text{for } t \geq 1$$

with the following constants:

$$m = (1-\tau)(1-p), \quad \kappa = \frac{(1+g)cb}{g}, \quad \gamma = \frac{(d-r)}{g}b\ , \quad \chi = (1-\tau)iw\ , \quad \vartheta = (1-w)k_E$$

$$z_{RI} = \frac{(1-\tau)\left(db + iw\left(a + b - \frac{d-r}{g}b\right)\right) + k_E\left((1-w)\left(a + b - \frac{d-r}{g}b\right) - \frac{(1+g)cb}{g}\right)}{1+g}$$

For residual income to grow at a constant rate (RSS), the following condition must be fulfilled:

(19a) $$(\chi + \vartheta)(\gamma R_0 - A_0) - k_E(\kappa R_0 - T_0) = 0$$

Or using the original parameters:

$$\left((1-\tau)iw+(1-w)k_E\right)\left(\frac{d-r}{g}bR_0-A_0\right)-k_E\left(\frac{1+g}{g}cbR_0-T_0\right)=$$

(19b) $$\left((1-\tau)iw+(1-w)k_E\right)\left((d-r)bR_0-gA_0\right)-k_E\left((1+g)cbR_0-gT_0\right)=$$

$$\left((d-r)bR_0-gA_0\right)-\frac{k_E}{\left((1-\tau)iw+(1-w)k_E\right)}\left((1+g)cbR_0-gT_0\right)=0$$

We can now more easily identify a case where (19b) holds. A sufficient condition is:

(19c) $gA_0=(d-r)bR_0$ and $gT_0=(1+g)cbR_0$

The first condition in (19c) is the 'depreciation link' seen for earnings (and dividend) steady state. It ensures that earnings grow at a constant rate. The second restriction concerns deferred taxes, and can be interpreted as the additional condition needed to get tax deferrals and book value of equity to grow at a constant rate.[14]

Summary and interpretations

We can now summarize the steady state conditions for the different valuation attributes:

Attribute	**Parameter restriction**
Free cash flow	*None*
Earnings	$gA_0=(d-r)bR_0$ or w=0
Dividends	$gA_0=(d-r)bR_0$ or w=0
Residual income	$\left((d-r)bR_0-gA_0\right)=\frac{k_E}{\left((1-\tau)iw+(1-w)k_E\right)}\left((1+g)cbR_0-gT_0\right)$ *sufficient:* $gA_0=(d-r)bR_0$ and $gT_0=(1+g)cbR_0$

As soon as input parameters are constant free cash flow grows at a constant rate. With all-equity financing, earnings and dividends require no further steady state conditions. For residual income, however, all-equity financing is not sufficient for steady state. Most companies have some debt, so the more general case (valid for all financing policies) is perhaps more interesting:

(20a) $gA_0=(d-r)bR_0$ *for earnings and dividends steady state,*

(21a) $gT_0=(1+g)cbR_0$ *additional condition sufficient for residual income steady state.*

A_0 (accumulated depreciation), R_0 (revenues) and T_0 (deferred taxes) are already given (principally by the forecasts from the explicit forecast period). We conjecture that one has some basis for deciding on the value of the growth rate, with a lower steady state bound being expected inflation. Furthermore, we find it reasonable that one has an idea about the amount of fixed assets necessary to generate a dollar of sales,[15] providing a value for the PPE-parameter *b*. In expression (20a) one is then left with $(d-r)$, i.e. the depreciation and retirements parameters; in expression (21a) the parameter governing deferred taxes, *c*, remains. Viewed in this manner, the conditions can be used to determine parameters (such as *r* or *c*) which may otherwise be elusive. The main importance of the conditions, however, is more intuitive. Expression (20a) establishes the steady state depreciation link between the balance sheet and the income statement. This becomes clearer after a reformulation:

$$\text{(20b)} \qquad g = \frac{(d-r)bR_0}{A_0}$$

Inserting expression (12) for t=1 yields:

$$\text{(20c)} \qquad g = \frac{A_1 - A_0}{A_0}$$

The right-hand side in (20c) is the growth rate in accumulated depreciation in year 1. This growth rate must be the same as the revenue growth rate g. How this can be achieved is given by expression (20a) or (20b): one can vary the forecast of depreciation expense in year 1 ($= dbR_0$) and/or of retirements in year 1 ($= rbR_0$). Operationally this can be done by varying any of the parameters involved. This exemplifies a general feature: The left-hand side in (20a) stands for the growth in the stock variable in the first post-horizon year, i.e., the growth in accumulated depreciation. The right-hand side describes the related flow variables, depreciation expense ($= dbR_0$) and retirements ($= rbR_0$) in the first year after the horizon. Similarly for (21b), the left-hand side is the growth in deferred taxes, the growth in the stock variable, and the right-hand side is the related flow variable. In both cases, the flows must be decided such that the related stocks grow at the revenue growth rate. Note that the intuition in terms of flows being decided to yield a certain growth rate for the stocks is quite general. Similar conditions will appear if we use different model specifications.[16]

3.3 Capital structure issues

In the prior sections we focused on the steady state conditions pertaining to the valuation attribute (the numerator in the continuing value). The discount rate k (which appears in the denominator of (2)) was not analyzed, even though the use of a continuing value formula also assumes that the discount rate is constant. In earnings-based valuation models, in the Residual Income model and in the Dividend Discount model the relevant discount rate is the *cost of equity*, which can vary with capital structure because of, e.g., expected costs of financial distress and agency costs.[17] Cost of equity also enters into the weighted average cost of capital (*wacc*) used in the Free Cash Flow model (where in addition the weights for the different costs of capital are given by the capital structure).

Capital structure is usually viewed to be in market value terms. In accounting-based valuation models we work with book values, however, and it is therefore of interest to examine the difference between capital structure in book value terms and capital structure in market value terms. Here, we will look at the difference between the book value debt ratio and the market value debt ratio.[18] Any differences between the two will be attributable to differences between the book asset value and the market asset value, since book value of debt equals market value by assumption. The market value debt ratio in an arbitrary year t in the steady state period will thus be:

$$\text{(22a)} \qquad \omega_t = \frac{D_t}{Assets_t \ \ \text{(at market value)}}$$

We need an expression for the market value of the asset side. The Free Cash Flow model provides this directly. Since our aim is to derive conditions for a constant market value capital structure, it is appropriate to use a constant discount rate in the derivation (in this case *wacc*):

(22b) $$\omega_t = \frac{D_t}{Assets_t} = \frac{w(aR_t + bR_t - A_t)}{\dfrac{FCF_{t+1}}{k_{WACC} - g}} = \frac{w(aR_t + bR_t - A_t)}{\dfrac{(1+g)R_t(m - z_{FCF})}{k_{WACC} - g}}$$

Substituting expressions (11) and (12) for R_t and A_t and rearranging:

(22c) $$\omega_t = \frac{w\left(a + b - \dfrac{d-r}{g}b\right)}{\dfrac{(1+g)(m - z_{FCF})}{k_{WACC} - g}} - \frac{w\left(A_0 - \dfrac{d-r}{g}bR_0\right)}{\dfrac{(1+g)^{t+1}R_0(m - z_{FCF})}{k_{WACC} - g}}$$

The first of the two terms in expression (22c) is the steady-state market value debt ratio. The second term is time-dependent and goes to zero only as *t* becomes large. Thus we need a parameter restriction to ensure that the market capital structure is constant. There are two possibilities: (i) all-equity financing (*w*=0); or (ii) $gA_0 = (d - r)bR_0$. These are exactly the same conditions that were derived in the previous section for valuation attributes that include financing (i.e., all attributes except *FCF*), so requiring a constant capital structure will only impose further conditions on the Free Cash Flow model – the same conditions as we already had for earnings and dividends. Not surprisingly, there is no escaping these conditions whichever model one chooses to work with: whereas *FCF* steady state may require fewer conditions, those conditions will instead appear in the discount rate, the denominator in the horizon value formula.

3.4 Balance sheet steady state (BSS)

From the results in Sections 3.2 and 3.3, it is straightforward to determine the conditions needed to ensure that each of the balance sheet items grows at a constant rate. This means that the balance sheet in relative terms remains unchanged. This would be another intuitive steady state definition.

Assets	***definition***	***steady state condition***
Net Working Capital	aR_t	*none*
Net Property, Plant and Equipment = Gross PPE – Accum. Depreciation	$bR_t - A_t$ where $A_t = A_{t-1} + (d-r)bR_{t-1}$	$gA_0 = (d-r)bR_0$

Debt and Equity	***definition***	***steady state condition***
Debt	$w(aR_t + bR_t - A_t)$	$gA_0 = (d-r)bR_0$
Deferred Taxes	$T_{t-1} + cbR_t$	$gT_0 = (1+g)cbR_0$
Equity	$(1-w)(aR_t + bR_t - A_t) - T_t$	$(d-r)bR_0 - gA_0 = (1+g)cbR_0 - gT$

If the parameters are constant, i.e. the company is in parametric steady state, it immediately follows that: net working capital will grow at the constant rate *g*, and gross property, plant and equipment will grow by *g*. If condition (20a) holds (linking depreciation stock and flow), then: accumulated depreciation will grow by *g*; net property, plant and equipment will grow by *g*; debt will grow by *g*. If also condition (21a) holds (linking stock and flow of tax deferrals), then: deferred taxes will grow by g, and book value of equity will grow by *g*.

To have BSS there are thus two necessary conditions: $gA_0 = (d-r)bR_0$ and $gT_0 = (1+g)cbR_0$. These two conditions have been discussed earlier, and one can see that there is a guiding principle at work: when establishing a valuation horizon, one should forecast the flows so that the corresponding stock variables grow at the revenue growth rate. One can also use this principle in reverse: one should define the valuation horizon as the point in time when it is deemed reasonable that the flows imply a growth rate in the stock variables equal to the revenue growth rate. This principle is quite general, and while we have shown it with ratios (the parameters) here, it would be applicable also to other models, where one directly forecasts balance sheet and income statement items without using ratios.

The BSS concept is useful because it ensures that the entire company remains similar year after year, that one has forecasted an internally consistent scenario that will prevail. In fact, we have a *general steady state* where the complete system of financial statements is in steady state: the valuation attributes, the income statement, and balance sheet items, as well as other relevant items (like capital expenditures and retirements) all grow at the same constant rate.

4 Does it all matter?

We will now try to gauge the empirical importance of steady state violations through a valuation case study and through observations on empirical large sample studies. The case study, which is a normal financial statement analysis type case, is the Swedish forestry company AssiDomän AB. The valuation is made as of January 1, 1998. Explicit forecasts are made for ten years, with each income statement and balance sheet item forecasted year by year. The time after that (i.e. after year 2008) is accounted for through a horizon value. In this paper we concentrate on the part of the valuation that concerns the issues involved in determining the horizon value.

From year 2008 and onwards the company is expected to have the following steady state parameter values: a=15.1%, b=130.9%, c=0.26%, d=4.7%, g=4.0%, i=7.5%, k_E=10.2%, p=81%, r=3.0%, τ=28% and w=25%. The forecasts made for the explicit forecast period give the initial values of the state variables: R_0=43.5, A_0=24.2 and T_0=3.9 (billion SEK). We will refer to this as the *base case*. The base case parameter values fulfill conditions (20a) and (21a), so we have a general steady state (all balance sheet items grow at a constant rate as do all valuation attributes). The equity value of the company when entering steady state at the end of year 2008 is 43.8 billion SEK.[19]

Now consider the retirements parameter, *r*. Assume that one does *not* use the steady state reasoning and condition (20a) to determine it, but rather looks at historical figures for the last few years. In the AssiDomän case the historical figure is 1.7% (compared to the steady state figure of 3.0%). Historical figures are often suggested as proxies for the long-term steady state value of different parameters, but setting r=1.7% means that the steady state condition (20a) is violated. Is this then of economic importance in the valuation? In AssiDomän (with r=1.7%), using the Residual Income model with a perpetuity formula yields an equity value of SEK 44.2 billion at the horizon. The Free Cash Flow model with a perpetuity yields an equity value of 55.9 billion at the horizon, i.e. around 25% higher. So the violation of the steady state condition is not only a theoretical problem. It has non-trivial economic consequences, consequences that are different for different valuation models. In this case, the violation has a much larger impact on the Free Cash Flow model than on the Residual Income model, and the Residual Income model is thus more 'robust' to this steady state violation. There are two reasons for this: First, the Residual Income model places less reliance than the Free Cash Flow model on the terminal value, so terminal value problems caused by steady state violations will have less impact on the Residual Income model. Second, accrual accounting places constraints on the behavior of residual income, whereas free cash flows are unconstrained by any reporting system. A mistake in forecasting the *r* parameter will impact the capital expenditure forecast, and all of capital expenditures affect the FCF amount used in the terminal value calculation while only a portion (the depreciation) will affect the residual income amount.

To further illustrate the effects of steady state violation we can look at other performance variables as well. Figure 1 has return on (beginning-of-year) book equity, i.e., a measure of profitability. Intuitively, one would expect profitability to be constant in steady state. This is indeed the case when *r*=3.0% (i.e. when condition (20a) is fulfilled). Not so when *r*=1.7%, however: when steady state is violated, then the performance changes after the horizon. And it is not only a one-time change; rather change becomes the predominant feature in the post-horizon period. The 1.7% case gives a profitability that increases from the base case level of 14.6% and approaches 34.0%.[20]

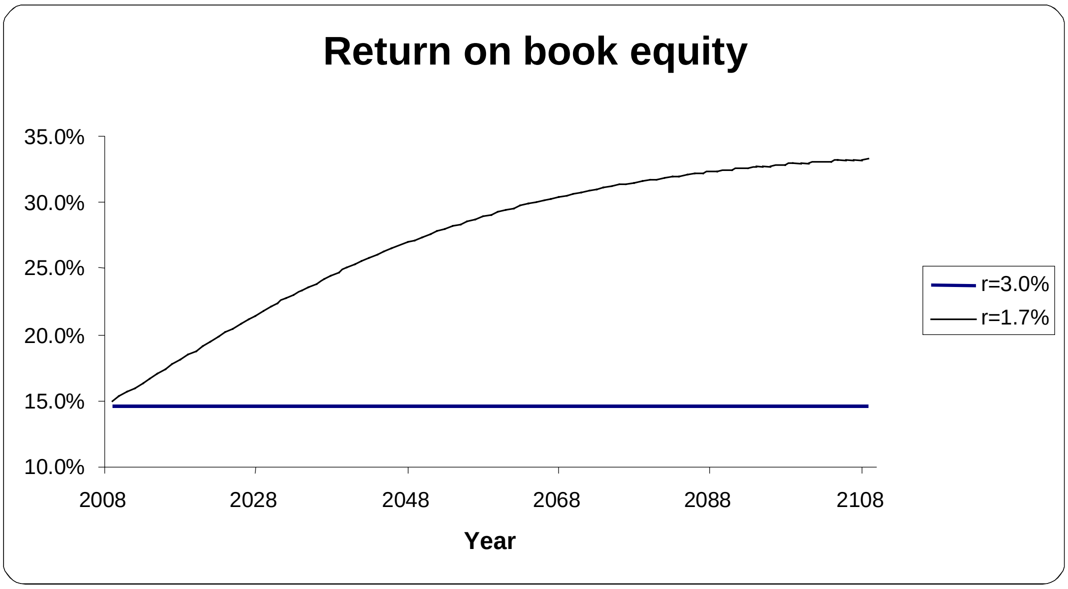


Figure 1 - Return on book equity

Return on book equity (ROE) is a particularly good indicator for our purposes. Intuitively, future profitability is central to valuation. More fundamentally, ROE includes the 'residuals' of the modeled financial statements: earnings (net income) is the ending item in the income statement and book value of equity is the ending item in the modeled balance sheet.[21] And this is the key: if the system (i.e., the modeled financial statements) is in steady state, then the 'residual' of each financial statement will be in steady state, i.e. both earnings and book value of equity will have the revenue growth rate. Consequently, the ratio between earnings and book value will be constant.

One should also note that it is the accrual accounting concept of return on book equity that serves as a check on steady state. Undoing the accruals (e.g., to calculate FCF) is not helpful in this respect. As we saw in section 3.2 free cash flow can grow at a constant rate even if steady state is violated, and as we saw earlier in this section the consequences of a steady state violation are likely to be much more severe for the Free Cash Flow model.

The Residual Income model and the Free Cash Flow model are both theoretically equivalent to the Dividend Discount model (given clean surplus). With the figures from the example above we have a situation where formally equivalent valuation models have the exact same input data and assumptions. But the violation of the steady state condition, introducing a changing profitability, has differential impact on the models causing the models to yield different values, in this case a difference of 25%.

There is some confusion in the debate about these issues. On the one hand most papers and textbooks agree that the models are formally equivalent. On the other hand, empirical studies find that the models yield vastly different value estimates (see, e.g., Penman and Sougiannis [1998] and Francis, Olsson and Oswald [2000]).[22] We claim that one of the most important and generally unrecognized reasons for such differences is the use of perpetuity formulas for the horizon value when steady state requirements are violated.

Consider Penman and Sougiannis [1998]. Their sample is basically the entire population of firms on Compustat. Furthermore, they use realizations to proxy for forecasts of different accounting items, which means that the sample is by definition internally consistent in the sense that all valuation attributes are derived from the same set of (realized) financial statements. Yet the models (using continuing-value type horizon values) yield completely different results. Using an explicit forecast period of four years, the Residual Income model has an average overestimation of 6% of actual price, whereas the Free Cash Flow model on average underestimates price by 76%.[23] Since the underlying forecasts are identical for the valuation models (i.e. realized financial statement data) and the models are theoretically equivalent, the main reason for the differences is the breakup of the infinite forecast horizon into an explicit forecast period and a horizon value. As shown earlier, the models will yield identical value estimates only if the steady state conditions hold at the horizon. This is obviously not the case in Penman and Sougiannis [1998] (nor in Francis, Olsson and Oswald [2000]), so the results in large sample empirical studies indicate that steady state violations are very important. As in the excerpt from the case study reported earlier in this section, it seems that the Residual Income model is far more robust than the Free Cash Flow model to such violations.

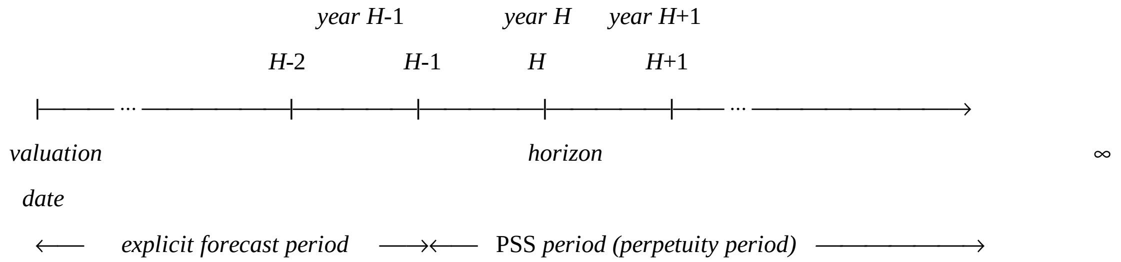
5 The Transition to Steady State

We now move to the question of *when* steady state is achieved. The analysis in Section 3 produced expressions for the valuation attributes and parameter restrictions that were functions of the initial values of the state variables, R_0, A_0 and T_0. That analysis was a pure steady-state analysis and as such was de-coupled from one very important implementation issue: the transition to steady state. Exact details about *how* the initial values of revenues, accumulated depreciation and deferred taxes should be specified were omitted. Intuitively, one would like to use the values of these state variables at H as initial values, i.e. take the values from the financial statements of the last year in the explicit forecast period and set $R_0 = R_H$, $A_0 = A_H$ and $T_0 = T_H$. Unfortunately, the lags in the model complicate the issue somewhat. To see why, consider accumulated depreciation as example. Accumulated depreciation in any year is defined as the preceding year's accumulated depreciation plus the current period's depreciation expense minus the book value of assets retired in the current period. In terms of our model:

(23) $$A_t = A_{t-1} + (d - r)bR_{t-1}$$

where bR_{t-1} represents gross PPE at t-1. If t= 0, then we need gross PPE at time (-1) which in general cannot be determined as $bR_{(-1)}$, because in year (-1), which does not belong to the PSS period, the value of b may differ from its steady state value. Similar problems apply for income statement items like depreciation expense and interest expense – in fact, for all items that are modeled using lagged values of some variable. In our model the maximum lag is one year, so the problem can be addressed by defining the horizon H as follows:

Explicit forecasts of full financial statements are made until and including year H, where year H is the first year with constant parameters. From the beginning of year H and on, all parameters defining the development of the company are constant, i.e. the company is in parametric steady state (PSS).



The initial values are then defined as:

(24) $R_0 = (1+g)R_{H-1} = R_H$

(25) $A_0 = A_{H-1} + (d-r)G_{H-1} = A_H$ (where $G_{H\text{-}1}$ is the forecasted gross PPE at the end of year H-1)

(26) $T_0 = T_{H-1} + cbR_0 = T_H$

Next, consider the valuation attributes. Remember that they in turn contain lagged values of the state variables. As a consequence the steady state expressions for the valuation attributes will only be valid from time H+1. This also means that VA_{H+1} will generally not equal $(1+g)VA_H$ and just taking the valuation attribute at the horizon (year H) and inserting it into a horizon value formula (as in expression (3)) can lead to severe valuation errors.[24] In summary, one year lags in both the state variables and in the valuation attributes imply that year H+1 is the appropriate time for measuring the inputs to the horizon value formula:

(2) $PV_H^{VA} = \dfrac{VA_{H+1}}{k-g}$

This means that the forecasting model (of financial statements) should include the first two years with constant parameters (years H and H+1). Financial statement analysis textbooks generally recognize that *valuation attributes* are lagged functions of the underlying forecasts of financial statements (see, e.g., Palepu, Bernard and Healy [1996, Ch. 6]). However, the *lags in the state variables* depend on the particulars of the forecasting model and are seldom discussed, even though it is very common in accounting-based forecasting to model at least some of the driving variables to include a lag.

It is time to summarize the steady state requirements: (i) the parameters should be constant from time H-1, (ii) the initial value conditions should be fulfilled at time H, and (iii) the horizon value calculation should be made using the valuation attribute at time H+1. Fortunately, this is all easier done than said: One normally sets up forecasts of all necessary items in a spreadsheet (income statement, balance sheet, perhaps *FCF* calculations, etc.). If one uses ratio forecasts for this task, then the first thing to remember is that the first year with constant ratios (parameters) should be used as year H, the horizon, where also the initial value conditions must be fulfilled. This can easily be done through, e.g., the goal seek function in Excel: the forecast of a ratio governing an income statement variable (or other flow item) should be made such that the corresponding stock variable (typically in the balance sheet) grows at the revenue growth rate. The same principle applies also if one does not use ratios for forecasting: then the direct forecasts of income statement items should be made such that the corresponding balance sheet items grow at the revenue growth rate. If one does not find this a reasonable assumption, then one should not use a horizon value at that time, but rather continue to make year-by-year forecasts until realistic values fulfil the steady state conditions. Thus, the steady state conditions can be useful in determining the appropriate horizon. The other thing to remember is that the ratios should be constant for two years (one year plus the maximum lag in the valuation model) before one uses a valuation attribute as numerator in a continuing value formula.

6 Summary and Concluding Remarks

This paper analyzes the time-series of forecasted income statements and balance sheets as a system of difference equations, and derives steady state conditions that bound the terminal value of the firm. The steady state conditions ensure that the company's forecasted performance is stable after the horizon. The basic assumption behind the steady state concept is that the expected development of the company, as described by the parameters, holds forever (in infinity). Internally inconsistent parameter determinations may result in an unrealistic forecast of the company's performance after the valuation horizon, which can significantly affect the value estimate calculated for the firm. The steady state conditions can thus serve both as a check against inconsistencies between parameter values, and as a toolkit for determining where the horizon should be set.

An immediate accounting interpretation of the steady state conditions is that they relate stock (balance sheet) to flow (income statement and related). The methodology used in this paper is general and can accommodate any forecasting model based on projected financial statements. The general principle is that all flow items in the post-horizon period should be decided so that the corresponding stock items grow at the constant revenue growth rate. This will yield a general steady state, which in turn guarantees that key variables like profitability remain constant after the valuation horizon. This is also a necessary condition for the Free Cash Flow model, the Dividend Discount model and the Residual Income model to yield identical results when implementing them with horizon values.

References

Brealey, R. and S. Myers. 1991. *Principles of Corporate Finance* (4th edition). New York: McGraw-Hill.

Copeland, T., T. Koller and J. Murrin. 1990 (1994). *Valuation: Measuring and Managing the Value of Companies*, 1st edition (2nd edition). New York: Wiley & Sons.

Fama, E. and K. French. 1997. Earnings, Investment, Dividends, and Debt. Working paper. University of Chicago and Yale University.

Francis, J., P. Olsson and D. Oswald. 2000. Comparing the Accuracy and Explainability of Dividend, Free Cash Flow and Abnormal Earnings Equity Value Estimates. *Journal of Accounting Research* 38/1, 45-70.

Kaplan, S., and R. Ruback. 1995. The Valuation of Cash Flow Forecasts: An Empirical Analysis. *Journal of Finance* 50/4: 1059-93.

Modigliani, F. and M. Miller. 1958. The Cost of Capital, Corporation Finance and the Theory of Investment. *American Economic Review* 48: 261-297.

Palepu, K., V. Bernard and P. Healy. 1996. *Business Analysis and Valuation: Using Financial Statements.* Cincinatti, Ohio: South-Western College Publishing.

Penman, S. 1997. A Synthesis of Equity Valuation Techniques and the Terminal Value Calculation for the Dividend Discount Model. Working paper. University of California at Berkeley.

Penman, S. and T. Sougiannis. 1998. A Comparison of Dividend, Cash Flow, and Earnings Approaches to Equity Valuation. *Contemporary Accounting Research* 15: 343-383.

Zhang, X.-J. 1998. Conservative Accounting and Equity Valuation. Working paper. University of California at Berkeley.

Appendix 1 - Derivations

Solution to system of difference equations

$$\begin{cases} R_t &= (1+g)R_{t-1} \\ A_t &= (d-r)bR_{t-1} + A_{t-1} \end{cases}$$

In matrix notation: $\mathbf{x_t} = \mathbf{A}\mathbf{x_{t-1}}$

where: $\mathbf{x_t} = \begin{pmatrix} R_t \\ A_t \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 1+g & 0 \\ (d-r)b & 1 \end{pmatrix}$

The roots of the characteristic equation are $\lambda_1 = 1+g$ and $\lambda_2 = 1$. **A** is diagonalized by **P**:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ \dfrac{(d-r)b}{g} & 1 \end{pmatrix}$$

$$\mathbf{P^{-1}AP} = \begin{pmatrix} 1+g & 0 \\ 0 & 1 \end{pmatrix}$$

Substituting $\mathbf{x_t} = \mathbf{Pu_t}$ and $\mathbf{x_{t+1}} = \mathbf{Pu_{t+1}}$ yields the system $\mathbf{u_{t+1}} = \mathbf{P^{-1}APu_t}$, the solution of which is:

$$\mathbf{u_t} = \begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix}$$

Substituting back yields the solution to the original system:

$$\mathbf{x_t} = \mathbf{Pu_t} = \begin{pmatrix} 1 & 0 \\ \dfrac{(d-r)b}{g} & 1 \end{pmatrix} \begin{pmatrix} c_1(1+g)^t \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1(1+g)^t \\ \dfrac{(d-r)b}{g} c_1(1+g)^t + c_2 \end{pmatrix}$$

and since the initial values are R_0 and A_0, the complete solution is:

$$\begin{cases} R_t &= (1+g)^t R_0 \\ A_t &= \dfrac{(1+g)^t - 1}{g}(d-r)bR_0 + A_0 \end{cases}$$

In the same way, the solution for T_t is derived.

Derivation of earnings expression - equation (14)

The earnings expression (14) is obtained by rearranging equation (4) and then substituting (11) and (12) for R_t and A_{t-1}. Note that the expression is only defined for $t \geq 1$ (since it contains t-1 values of state variables that are defined with initial values at time zero).

$$X_t = (1-\tau)[R_t + pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})]$$

$$= (1-\tau)(1-p)R_t - (1-\tau)(db + iw(a+b))R_{t-1} + (1-\tau)iwA_{t-1}$$

$$= R_t\left[(1-\tau)(1-p) - \frac{(1-\tau)(db+iw(a+b))}{1+g}\right] + (1-\tau)iw\left(A_0 + \frac{(d-r)b}{g}(R_{t-1} - R_0)\right)$$

$$= R_t\left[(1-\tau)(1-p) - \frac{(1-\tau)(db+iw(a+b)) - (1-\tau)iw\frac{(d-r)b}{g}}{1+g}\right] + (1-\tau)iw\left(A_0 - \frac{(d-r)b}{g}R_0\right)$$

$$= (1+g)^t R_0\left[(1-\tau)(1-p) - \frac{(1-\tau)\left(db + iw\left(a+b-\frac{(d-r)b}{g}\right)\right)}{1+g}\right] - (1-\tau)iw\left(\frac{(d-r)b}{g}R_0 - A_0\right)$$

This yields equation (14) for $t \geq 1$:

(14) $$X_t = (1+g)^t R_0 (m - z_X) - \chi(\gamma R_0 - A_0)$$

with the following constants:

$$m = (1-\tau)(1-p), \quad z_X = \frac{(1-\tau)\left(db + iw\left(a+b-\frac{d-r}{g}b\right)\right)}{1+g}$$

$$\chi = (1-\tau)iw, \quad \gamma = \frac{(d-r)}{g}b$$

Derivation of free cash flow expression - equation (16)

The derivation is carried out by rearranging expression (5) and making subsequent use of expressions (8) and (10):

$$FCF_t = (1-\tau)(R_t - pR_t - dbR_{t-1}) + dbR_{t-1} + (T_t - T_{t-1}) - (aR_t - aR_{t-1}) - (bR_t - bR_{t-1} + rbR_{t-1})$$

$$= R_t[(1-\tau)(1-p) - a - b] + R_{t-1}[a + (1-r)b + \tau db] + T_t - T_{t-1}$$

$$= R_t\left[(1-\tau)(1-p) - \frac{(1+g)(a+b) - a - (1-r)b - \tau db}{1+g}\right] + cbR_t$$

$$= R_t\left[(1-\tau)(1-p) - \frac{-\tau db - (1+g)cb + ga + (g+r)b}{1+g}\right]$$

By using expression (11), we obtain expression (16) for $t \geq 1$:

(16) $$FCF_t = (1+g)^t R_0 (m - z_{FCF})$$

with the following constants:

$$m = (1-\tau)(1-p), \quad z_{FCF} = \frac{-\tau db - (1+g)cb + ga + (g+r)b}{1+g}$$

Derivation of dividend expression - equation (17)

The dividend expression (17) is derived by rearranging expression (6), and using expressions (10), (11) and (12):

$$DIV_t \quad = \quad (1-w)(aR_{t-1}+bR_{t-1}-A_{t-1})-T_{t-1}+(1-\tau)(R_t-pR_t-dbR_{t-1}-iw(aR_{t-1}+bR_{t-1}-A_{t-1}))$$

$$-[(1-w)(aR_t+bR_t-A_t)-T_t]$$

$$= \quad R_t[(1-\tau)(1-p)-(1-w)(a+b)]+R_{t-1}[(1-w)(a+b)-(1-\tau)(db+iw(a+b))]$$

$$+A_t(1-w)+A_{t-1}[(1-\tau)iw-(1-w)]+T_t-T_{t-1}$$

$$= \quad R_t\left[(1-\tau)(1-p)-\frac{(1+g)[(1-w)(a+b)]-[(1-w)(a+b)-(1-\tau)(db+iw(a+b))]}{1+g}\right]$$

$$+A_t(1-w)+A_{t-1}[(1-\tau)iw-(1-w)]+cbR_t$$

$$= \quad R_t\left[(1-\tau)(1-p)-\frac{(1+g)[(1-w)(a+b)-cb]-[(1-w)(a+b)-(1-\tau)(db+iw(a+b))]}{1+g}\right]$$

$$+A_t(1-w)+A_{t-1}[(1-\tau)iw-(1-w)]$$

$$= \quad R_t\left[(1-\tau)(1-p)-\frac{-(1-\tau)db-(1+g)cb+g(1-w)(a+b)+(1-\tau)iw(a+b)}{1+g}\right]$$

$$+\frac{(1+g)^t-1}{g}(d-r)bR_0(1-w)+A_0(1-w)$$

$$+\frac{(1+g)^{t-1}-1}{g}(d-r)bR_0[(1-\tau)iw-(1-w)]+A_0[(1-\tau)iw-(1-w)]$$

$$= \quad R_t\left[(1-\tau)(1-p)-\frac{-(1-\tau)db-(1+g)cb+g(1-w)(a+b)+(1-\tau)iw(a+b)}{1+g}\right]$$

$$+R_t\frac{(d-r)b}{g}(1-w)+R_t\frac{(d-r)b}{g(1+g)}[(1-\tau)iw-(1-w)]-R_0\frac{(d-r)b}{g}[(1-\tau)iw]+A_0(1-\tau)iw$$

$$= \quad R_t\left[(1-\tau)(1-p)-\frac{-(1-\tau)db-(1+g)cb+g(1-w)(a+b)+(1-\tau)iw(a+b)}{1+g}\right]$$

$$+R_t\left[\frac{(1+g)\frac{(d-r)b}{g}(1-w)+\frac{(d-r)b}{g}[(1-\tau)iw-(1-w)]}{1+g}\right]-R_0\frac{(d-r)b}{g}[(1-\tau)iw]+A_0(1-\tau)iw$$

$$=R_t\left[(1-\tau)(1-p)-\frac{-(1-\tau)db-(1+g)cb+g[(1-w)(a+b)]+(1-\tau)iw(a+b)-[g(1-w)+(1-\tau)iw]\frac{(d-r)b}{g}}{1+g}\right]$$

$$-(1-\tau)iw\left(\frac{(d-r)b}{g}R_0-A_0\right)$$

Expression (17) is obtained for $t \geq 1$:

$$(17) \qquad DIV_t = (1+g)^t R_0 (m - z_{DIV}) - \chi(\gamma R_0 - A_0)$$

with the following constants:

$$m = (1-\tau)(1-p)$$

$$z_{DIV} = \frac{-\tau db - (1+g)cb + ga + (g+r)b - gw\left(a + b - \frac{d-r}{g}b\right) + (1-\tau)iw\left(a + b - \frac{d-r}{g}b\right)}{1+g}$$

$$\chi = (1-\tau)iw\,, \quad \gamma = \frac{(d-r)}{g}b$$

Derivation of residual income expression - equation (18)

The residual income expression (18) is obtained by rearranging equation (7) and then substituting expressions (11), (12) and (13):

$$RI_t = (1-\tau)[R_t + pR_t - dbR_{t-1} - iw(aR_{t-1} + bR_{t-1} - A_{t-1})] - k_E[(1-w)(aR_{t-1} + bR_{t-1} - A_{t-1}) - T_{t-1}]$$

$$= (1-\tau)(1-p)R_t - [(1-\tau)db - [(1-\tau)iw + (1-w)k_E](a+b)]R_{t-1} + [(1-\tau)iw + (1-w)k_E]A_{t-1} + k_E T_{t-1}$$

$$= R_t\left[(1-\tau)(1-p) - \frac{(1-\tau)(db + iw(a+b)) + k_E(1-w)(a+b)}{1+g}\right]$$

$$+ [(1-\tau)iw + k_E(1-w)]\left(A_0 + \frac{(d-r)b}{g}(R_{t-1} - R_0)\right) + k_E\left[T_0 + \frac{cb}{g}R_t - \frac{cb}{g}R_0 - cbR_0\right]$$

$$= R_t\left[(1-\tau)(1-p) - \frac{(1-\tau)\left(db + iw\left(a + b - \frac{(d-r)}{g}b\right)\right) + k_E\left[(1-w)\left(a + b - \frac{(d-r)}{g}b\right) - \frac{(1+g)cb}{g}\right]}{1+g}\right]$$

$$- [(1-\tau)iw + k_E(1-w)]\left(\frac{(d-r)b}{g}R_0 - A_0\right) - k_E\left[\frac{(1+g)cb}{g}R_0 - T_0\right]$$

This yields equation (18) for $t \geq 1$:

$$(18) \qquad RI_t = (1+g)^t R_0 (m - z_{RI}) - (\chi + \vartheta)(\gamma R_0 - A_0) - k_E(\kappa R_0 - T_0)$$

with the following constants:

$$m = (1-\tau)(1-p)$$

$$z_{RI} = \frac{(1-\tau)\left(db + iw\left(a + b - \frac{d-r}{g}b\right)\right) + k_E\left((1-w)\left(a + b - \frac{d-r}{g}b\right) - \frac{(1+g)cb}{g}\right)}{1+g}$$

$$\chi = (1-\tau)iw\,, \quad \vartheta = (1-w)k_E\,, \quad \gamma = \frac{(d-r)}{g}b\,, \quad \kappa = \frac{(1+g)cb}{g}$$

Appendix 2 - Steady state conditions for alternative model specifications

This appendix provides steady state conditions for two alternative model specifications that have been suggested in the literature. The alternative specifications differ from the one used in the main text in how the investments in PPE are modeled. Previously, *gross* PPE has been modeled as a constant percentage of revenues, denoted by the parameter b (as in Copeland, Koller and Murrin [1990]). In the first alternative here, following Palepu, Bernard and Healy [1996] and Copeland, Koller and Murrin [1994], it is *net* PPE that is instead modeled as a percentage of revenues, n. The second alternative, also suggested in Copeland, Koller and Murrin [1994], models the capital expenditures as a percentage of revenues, e. For these alternatives the following differs from the original modeling in section 2:

<u>Alternative 1 (Palepu, Bernard and Healy [1996], Copeland, Koller and Murrin [1994]):</u>

Net Property, Plant and Equipment:	nR_t
Gross Property, Plant and Equipment	$nR_t + A_t$
Accumulated Depreciation:	$A_t = A_{t-1} + (d-r)(nR_{t-1} + A_{t-1})$
Deferred taxes:	$T_t = T_{t-1} + c(nR_t + A_t)$

<u>Alternative 2 (Copeland, Koller and Murrin [1994]):</u>

-- *G_t, gross PPE, is an additional state variable* --

Net Property, Plant and Equipment:	$G_t - A_t$
Gross Property, Plant and Equipment	$G_t = G_{t-1} + eR_t - rG_{t-1}$
Accumulated Depreciation:	$A_t = A_{t-1} + (d-r)G_{t-1}$
Deferred taxes:	$T_t = T_{t-1} + cG_t$

By using the same line of reasoning as for the original specification, we get the following conditions for the alternative specifications:

<u>Alternative 1:</u>

Attribute	**Parameter restriction**
Free cash flow	$gA_0 = (d-r)(nR_0 + A_0)$
Earnings	$gA_0 = (d-r)(nR_0 + A_0)$
Dividends	$gA_0 = (d-r)(nR_0 + A_0)$
Residual income	$gA_0 = (d-r)(nR_0 + A_0)$ <u>and</u> $gT_0 = (1+g)c(nR_0 + A_0)$

Alternative 2:

Attribute	Parameter restriction
Free cash flow [25]	$(g+r)G_0 = e(1+g)R_0 \;\Leftrightarrow\; gG_0 = e(1+g)R_0 - rG_0$
Earnings	$gA_0 = (d-r)e\dfrac{(1+g)}{(g+r)}R_0$
Dividends	$gA_0 = (d-r)e\dfrac{(1+g)}{(g+r)}R_0$
Residual income	$gA_0 = (d-r)e\dfrac{(1+g)}{(g+r)}R_0$ <u>and</u> $gT_0 = (1+g)ce\dfrac{(1+g)}{(g+r)}R_0$

In both alternative cases, not even FCF grows at a constant rate, unless an initial value condition is fulfilled. Moreover, all-equity financing is no longer a sufficient condition for having a constant growth in earnings and dividends. In fact, both these specifications (which are the advocated by the most recent textbooks in the area) are dangerous in this respect: continuing value calculations (regardless of which valuation model is used) will be incorrect, unless the initial value condition is fulfilled.

Taking capital structure into consideration, thus completing the valuation models, the above tables give the correct conditions, except for FCF in Alternative 2, where both $(g+r)G_0 = e(1+g)R_0$ <u>and</u> $gA_0 = (d-r)e\dfrac{(1+g)}{(g+r)}R_0$ have to be fulfilled.

---

[1] Penman (1997) provides a horizon value approach focusing on the measurement error of the accounting, where formula (2) arises as special case. The underlying requirement is that the growth rate of the measurement error is constant between any subsequent set of S years (beyond H). In practice, the problem is to determine this S-year growth rate. It can be inferred from the projected constant growth rate of residual income beyond the horizon; the approach still requires steady state assumptions. In this paper we stick to the standard application with 1-year period length (and 1-year growth rates), but note that the analysis could be performed using S-year cycles instead without any effect on the intuition of our results.

[2] Horizon value problems are likely to be most severe in valuation models that place the largest weight on the horizon values. For a given company, the free cash flow horizon value is generally larger than the dividends (or earnings) horizon value, which in turn is larger than the residual income horizon value.

[3] The parameters are initially restricted as follows: $b>0$; $c\in[0, 1]$; $d\in(0, 1]$; $g\in(0, k)$; $i>0$; $k>0$; $p\in(0, 1)$; $r\in(0, 1]$; $\tau\in[0, 1)$; $w\in[0, 1)$. Net working capital is thus unrestricted. Gross PPE must be positive. The change in deferred taxes can be zero but not negative in the perpetuity period, also new tax deferrals during a year cannot exceed gross PPE. At least some fraction of the prior year's physical capital must be depreciated, but depreciation cannot exceed 100%. The growth rate must be positive but smaller than the discount rate (since the company value would otherwise be infinite). The interest rate is positive, as is the discount rate. The operating profit margin is positive but less than 100%. Some fraction of preceding year's physical assets must be retired, but one cannot retire more than 100%. The tax rate can be zero but must be smaller than 100%. Debt can be zero, but the company is not allowed to be 100% debt financed.

[4] Cost of equity, k_E, for dividends, earnings and residual income; weighted average cost of capital (*wacc*) for free cash flow.

[5] Interest expense is calculated based on beginning-of-year debt.

[6] Note that a company cannot make losses in perpetuity, so earnings before taxes are non-negative in steady state. This makes the modelling of taxes straightforward.

[7] Deferred taxes are explicitly excluded from the debt governed by w. The reason is that tax deferrals are linked to the firms operations and thus not really an item included in financing decisions. Conceptually, one may even think about deferred taxes as a negative item on the asset side.

[8] Modelling deferred taxes may seem something of an overkill in a stylized model like this. They serve a special purpose, however. In many countries tax accounting is distinct from GAAP accounting in some aspects. Thus we want to model the difference between the provision for taxes in the (GAAP) income statement and actual taxes paid. In a particular year this difference equals the change in deferred taxes on the balance sheet. Exactly how this change should be specified depends on a country's tax rules. Here, we model deferred taxes as they are defined in Copeland, Koller and Murrin [1990], noting that much more exact country-specific models can be applied.

[9] (1 – tax rate) (revenues – op. exp. – depr) + depr + change in def. taxes – change in net working cap. – capital expenditures.

[10] Residual income equals earnings minus the cost of capital times the lagged book value of equity.

[11] Palepu, Bernard and Healy [1996] discuss growth rate assumptions, i.e., the value of g in our model. In particular, they argue the insensitivity of company value to sales growth assumptions if one assumes a competitive equilibrium. Note, however, that although the company value may – in some circumstances – be insensitive to the exact value of the growth rate g, it will still be necessary to have the *same* growth rate, g, for the different attributes.

[12] As mentioned in the introduction, the typical situation is that full financial statements are modeled explicitly for a limited number of years, 'the explicit forecast period'. The values of variables at the end of this period – at the horizon – are in essence the initial values of the post-horizon period (see Section 5 for more exact implementation details).

[13] Because free cash flow is by definition independent of financing, the all-equity financing restriction is irrelevant. Free cash flow also does not include depreciation, so depreciation issues become irrelevant (except for their influence on taxes paid, but that effect is included in the z_{FCF} constant).

[14] For earnings steady state an alternative sufficient condition was all-equity financing (w=0). This will not suffice for residual income to grow at a constant rate, however. With all-equity financing expression (19b) reduces to: $(d-r)bR_0 - gA_0 = (1+g)cbR_0 - gT_0$.

[15] Based on the analyst's knowledge of the company, comparable companies, etc.

[16] See Appendix 2 for the resulting conditions for two alternative model specifications from the literature.

[17] Even in a Modigliani and Miller [1958] world the cost of equity is a function of leverage (while the weighted average cost of capital is constant). In a setting with corporate taxes, both the cost of equity and the weighted average cost of capital can vary with leverage.

[18] There is no preferred stock in the model, so we need only look at the debt ratio (since the equity ratio is simply one minus the debt ratio).

[19] Calculated using, e.g., the Residual Income model: $\textit{Equity value}_0 = \textit{Book equity}_0 + RI_1/(k_E - g) =$ / using the residual income expression (17) / $= 25.6 + 1.13/(0.102 - 0.040) = 43.8$ (billion SEK). Since the steady state conditions are fulfilled, the Free Cash Flow model and the Dividend Discount model give exactly the same result. See, e.g., Zhang [1998] for an analytical proof. Note that while that paper points to the importance of the steady state concept in this respect, the purpose of our paper is to develop a general methodology to make steady state operational.

[20] Figure 1 is derived by using the initial values of R_0, A_0 and T_0 as given from the explicit forecast period and using the constant parameters to derive all income statement and balance sheet variables year by year after the horizon. This in turn makes it possible to calculate ROE year by year.

[21] Remember that while in actual, realized, financial statements these things are defined simultaneously through double-entry bookkeeping, in valuation we deal with forecasts of both income statement and balance sheet, where earnings are the residual item of the income statement and book value of equity is the residual item of the balance sheet.

[22] We stress that this is not a criticism. Both Penman and Sougiannis [1998] and Francis, Olsson and Oswald [2000] are perfectly representative of common practice in this regard (which is also their purpose).

[23] Penman and Sougiannis [1998, table 1]. The differences are vast regardless of the length of the explicit forecast period.

[24] In the AssiDomän AB case, this would lead to an overpricing by around 22%.

[25] The interpretation of this condition is that capital expenditures ($= e(1+g)R_0$) minus retirements ($= rG_0$) in the first year of the steady state period must be decided such that gross PPE grows at the revenue growth rate.